A global Touring of Art Exhibition of Computational Discrete Global Geometric Structures

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Abstract： In 2024, the Computational Discrete Global Geometric Structures Laboratory launched a touring art exhibition covering numerous universities and research institutions across China. The exhibition focuses on cutting-edge geometric topology theories and concepts, with a particular emphasis on various global geometric structures. Leveraging images and videos generated by new computer algorithms, original code, and computer graphics rendering technology, the national touring art exhibition transforms abstract intrinsic geometric structures into intuitive visual presentations, which are displayed in the form of large-scale posters. The innovation of the exhibition content lies in its embodiment of intrinsic global geometric topology concepts developed by mathematicians over the past few decades.To date, the traveling exhibition has been held at more than ten universities and is still ongoing. The entire tour is expected to span ten years and cover 100 universities. Through this innovative art exhibition format, teachers and students from different majors at universities across China have gained a direct understanding of geometric topology concepts that they rarely encountered before, stimulating their research interest and laying a foundation for in-depth exploration of cutting-edge geometric topology theories and their applications. Furthermore, it paves the way for interdisciplinary integration in various science and engineering disciplines—such as mechanics, mechanical engineering, computer science, physics, and materials science—through the application of cutting-edge geometric topology theories. This national traveling art exhibition has also pioneered a brand-new genre in the art field: "Global Geometric Structure Geometric Topology Art."

Key Words： Super Structured Quad Mesh, Harmonic Foliation, Global Geometric Structures, Art

（Figure 1： Harmonic foliation， generated by Hui Zhao with the software Geometric .）

1 Overview

Mathematicians use abstraction as a tool to break free from the constraints of physical conditions such as external equipment and experiments; their only limitations are time and energy. Unburdened by such constraints, they can explore broader fields and construct new mathematical structures that are not restricted by external conditions.

As a powerful tool, abstraction has two sides. However, precisely because of its abstract nature, the new achievements, new worlds explored, and new structures built by mathematicians are often difficult for people who are accustomed to relying on concrete examples for reference to understand and appreciate easily.

Vision is the primary channel through which humans receive information. Transforming abstract mathematical concepts into visual forms can greatly facilitate the rapid understanding and experience of new fields pioneered by mathematicians, and accelerate the diffusion of new structures developed by mathematicians into the field of engineering technology.

In the past, mathematicians hand-drew a large number of sketches to illustrate the geometric topology concepts they studied. However, hand-drawn sketches cannot fully demonstrate these concepts from various perspectives in depth. We use computer graphics rendering technology to create a comprehensive exhibition featuring 3D space and photorealistic rendering, which enhances the visual immersion and thus elevates the presentation to an artistic level.

To advance geometric topology concepts from hand-drawn sketches to computer visualization, it is first necessary to perform numerical calculations on relevant geometric concepts through algorithm design. With the development of computer science, research in this field has become increasingly mature in recent years; more and more geometric topology structures that were once difficult to quantify can now be captured by algorithms, as shown in Figure 1 (Harmonic Foliations).

Just as mathematical structures such as Fourier transforms—proposed by mathematicians hundreds of years ago — gradually permeated the field of engineering technology through computation; the mathematical theories from Newton’s era are now widely and maturely applied in today’s engineering technology. In contrast, the new geometric theories developed by the mathematics community in recent decades are still in the historical process of diffusing into application fields.

Based on our 30 years of accumulated work in algorithms, rendering, and applications, and with the strong support of teachers and students from multiple universities, we have held the "National Touring Art Exhibition of computational discrete Global Geometric Structures" at more than ten universities and research institutions since 2024. The exhibition focuses on showcasing some of the global geometric structures we have studied, and is still ongoing. It has aroused strong interest among visiting teachers and students in cutting-edge geometric topology theories, computer graphics rendering, algorithm design, and industrial technology applications.

Different from common activities themed around "the beauty of mathematics" or "the beauty of geometry," the innovation of this national traveling art exhibition mainly lies in being the first in the world to present the concept of "intrinsic global geometric structures" using computer graphics rendering technology.

(Figure 2: On-site photo of the exhibition at the School of Mathematics, Taiyuan University of Technology.)

2 The Content of the Exhibition

In previous research, to learn, understand, master, explore, and apply geometric topology concepts—while also designing related algorithms—we rendered tens of thousands of images with various rendering effects. Through repeated comparison of these images, we gradually gained insight into the in-depth meanings of relevant geometric topology concepts and theories.

The primary purpose of creating images via computers is to design computer algorithms for solving related industrial and technical problems, particularly mesh-related technologies. Through meticulous rendering, abstract mathematical concepts can be easily understood, which in turn enables the solution of technical challenges that cannot be addressed using common mathematical tools.

From these images, the national traveling art exhibition selected more than 50 representative works, all focusing on global geometric structures and their applications. These works collectively present the algorithmic visualization research results of cutting-edge geometric topology theories and concepts, and vividly demonstrate some geometric structures on 2D curved surfaces in particular.

The geometric topology categories covered in these images include the following:

(1) Ribbon-Graph

(2) Square-Tiling

(3) Non-harmonic Foliations

(4) Harmonic Foliations

(5) Harmonic Foliations with Poles

(6) Non-harmonic Foliations with Poles

(7) Harmonic Foliations on Surfaces with Boundaries

(8) Holomorphic Quadratic Differentials

(9) Meromorphic Quadratic Differentials

(10) Differential 1-Forms

(11) Holomorphic 1-Forms

(12) Discrete Calabi Flow

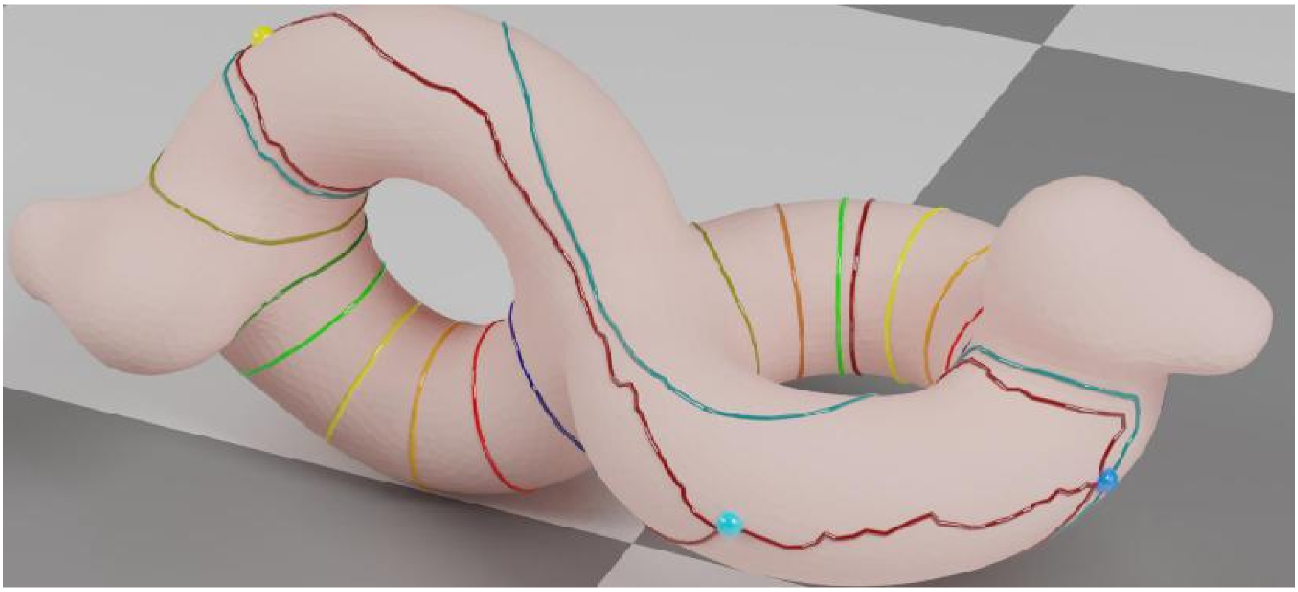
(13) Normal Flow

(14) Parameterization Applications

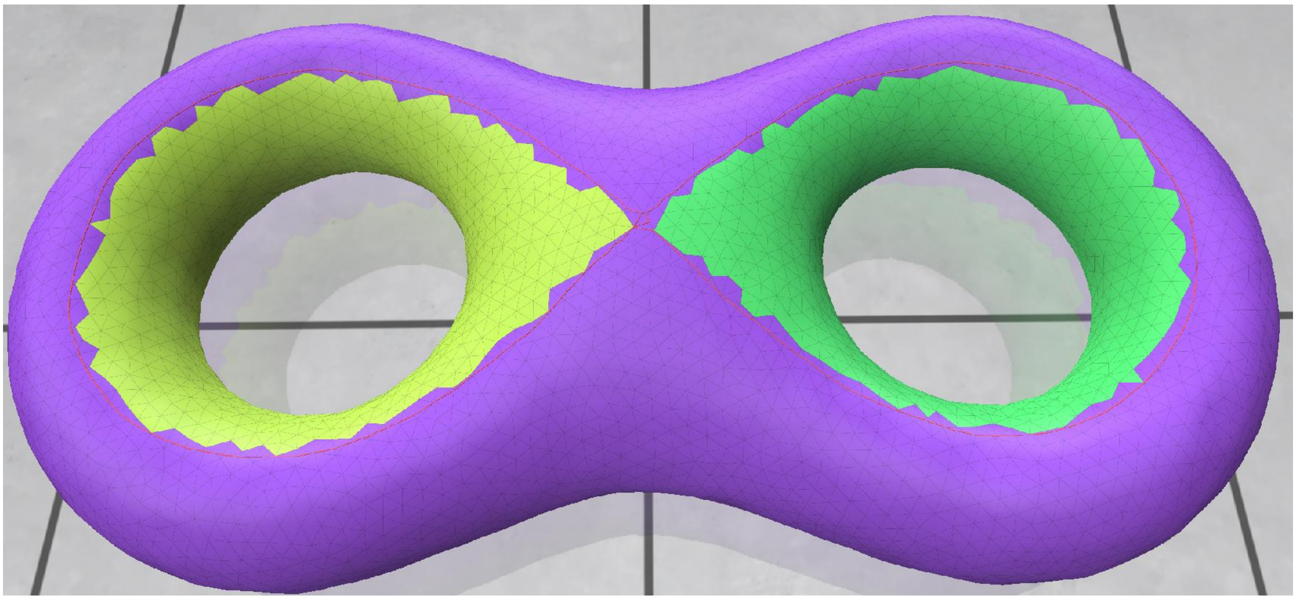
(15) Mesh Generation Applications

(16) Etc.

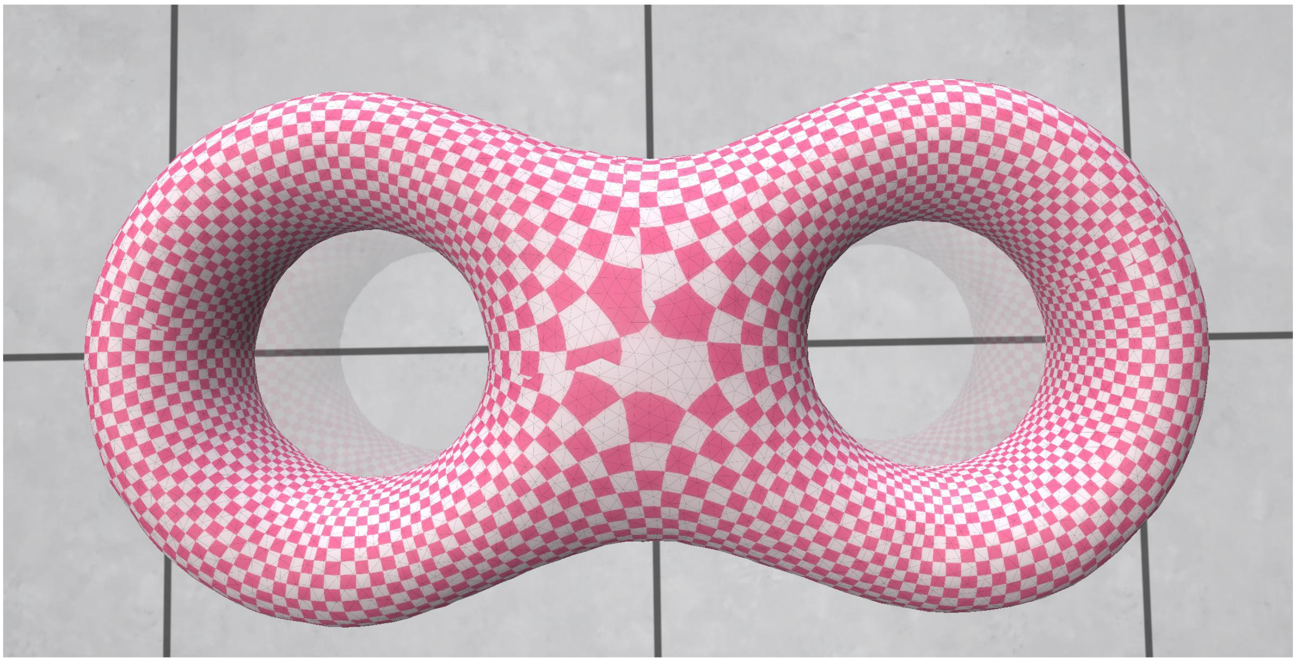
Since the meanings of the aforementioned mathematical concepts and theories are rarely encountered in daily study or work, it is difficult to quickly understand them directly from the literal meaning of their mathematical definitions. Therefore, the national traveling art exhibition we proposed uses visual means to allow audiences to gain an intuitive perception of these abstract concepts at first glance. A selection of representative images from the exhibition is presented below; for more images, please refer to the appendix.



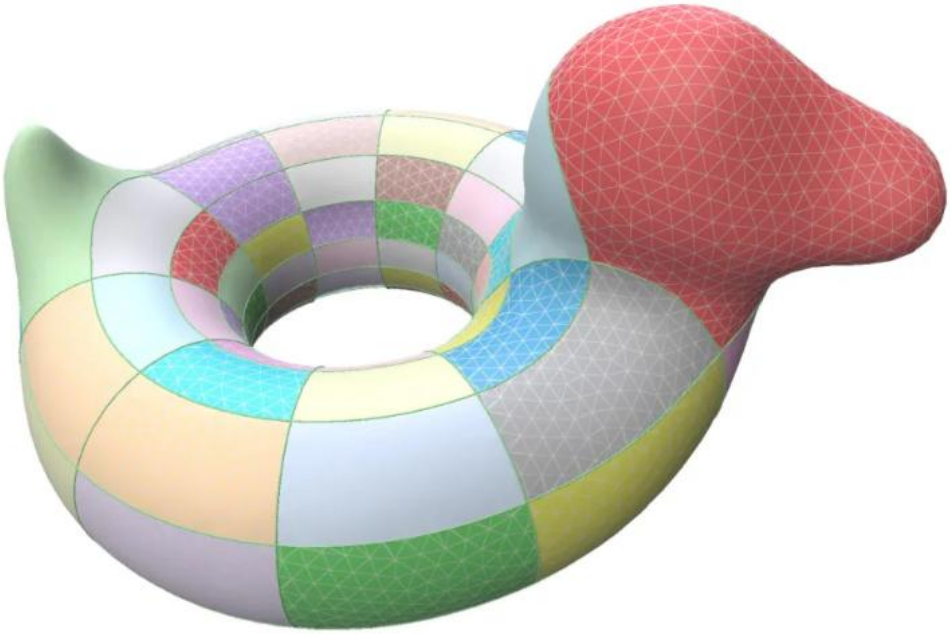
(Figure 3： Ribbon graph, generated by our software Geometric..)



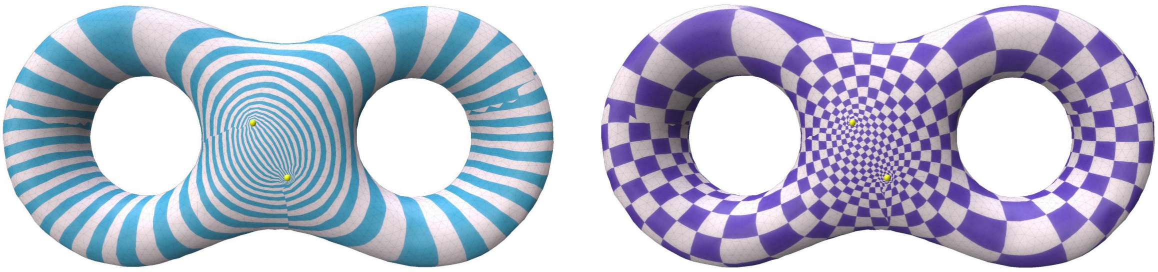
(Figure 4： Non-harmonic foliation, generated by our software Geometric.)



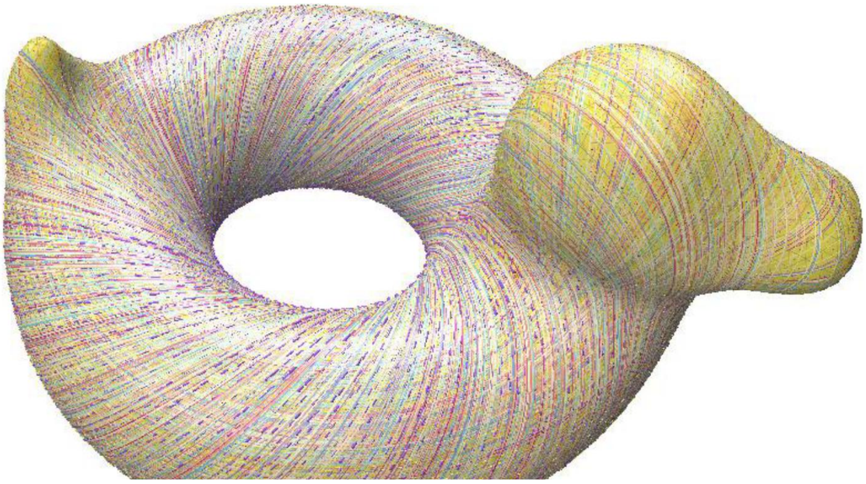
（Figure 4： Holomophic quandratic differential, generated by our software Geometric.）



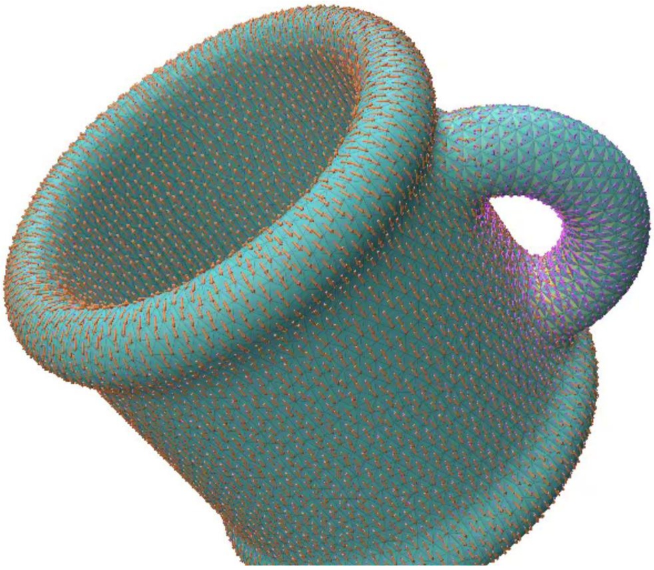
(Figure 5： Square-tiling, generated by our software Geometric.)



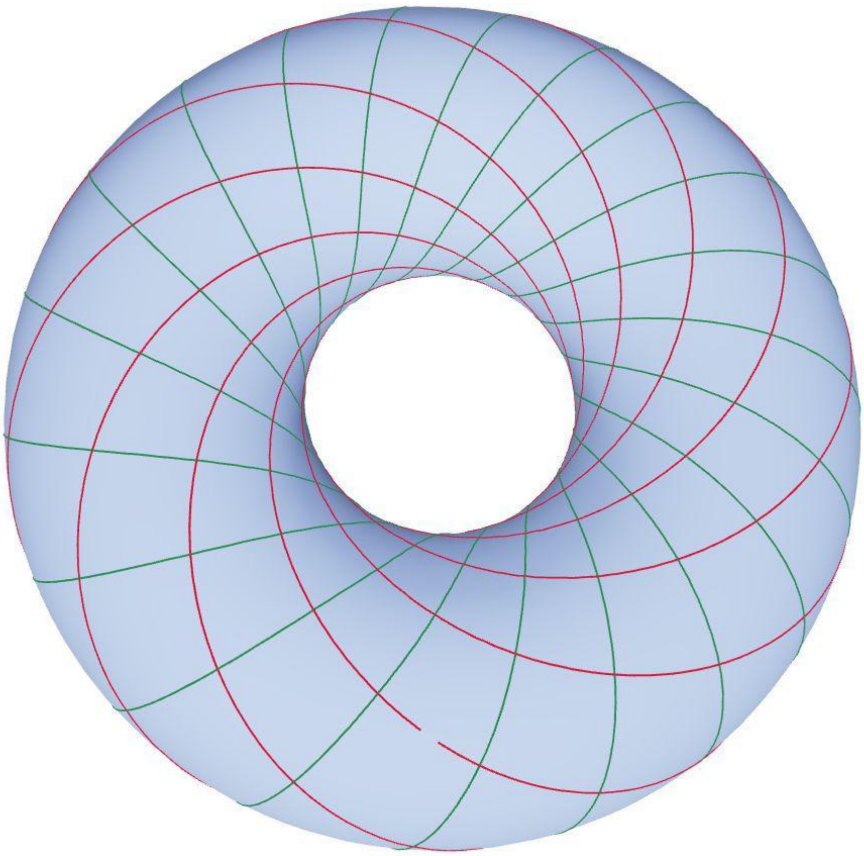
(Figure 6: Left is harmonic foliation with 2 poles, right is their corresponding meromorphic holomophic quadratic differentials, generated by our software Geometric.)



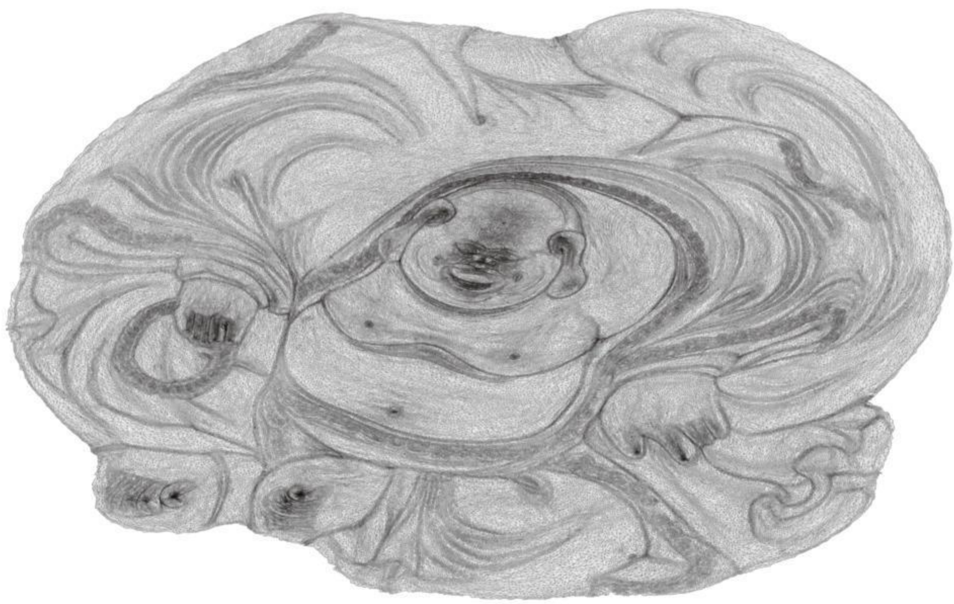
(Figure 7: Differential 1-form, generated by our software Geometric.)



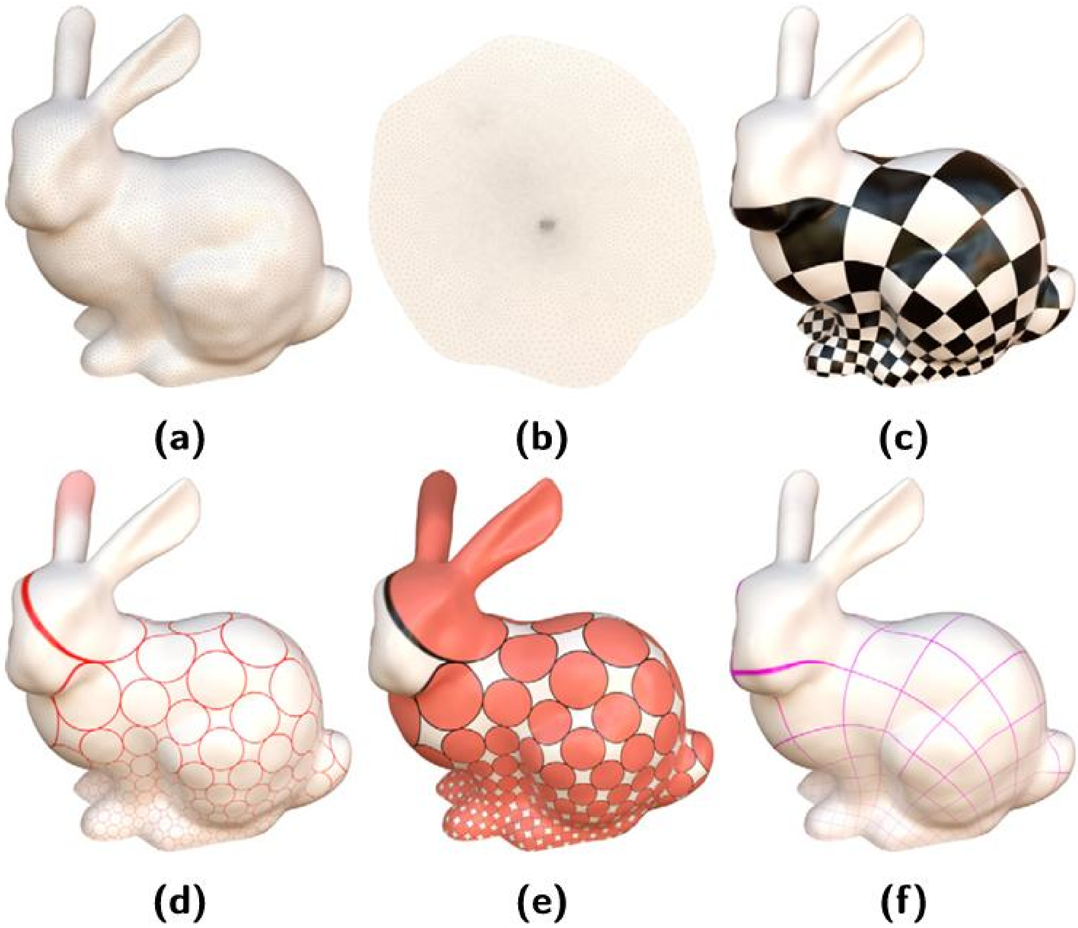
(Figure 8: Vector field, generated by our software Geometric.)



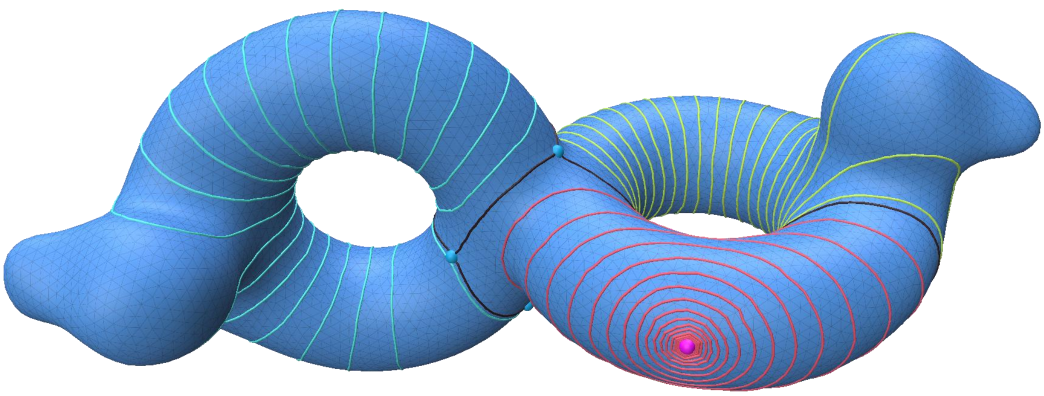
（Figure 9： Super structured quad mesh, generated by our software Geometric.）



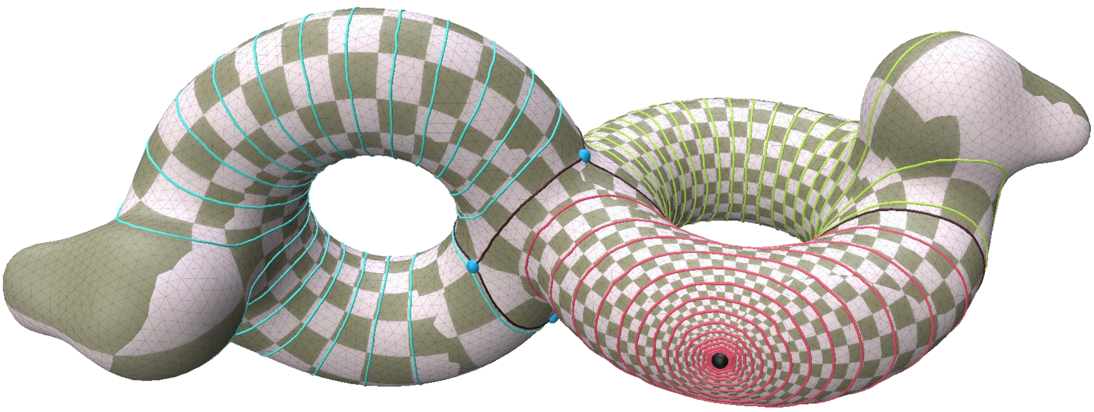
（Figure 10： Parameterization by unit normal flow, generated by our software Geometric.）



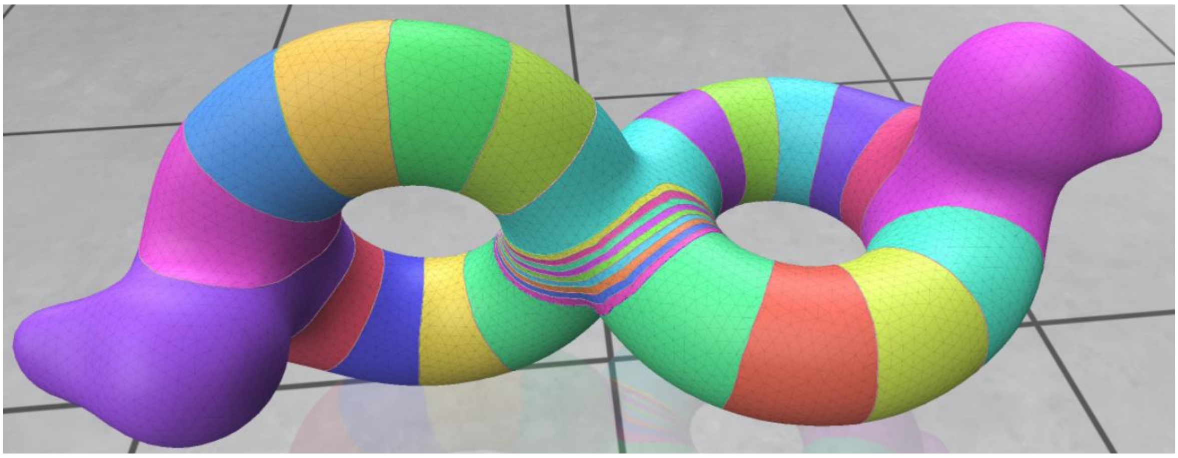
(Figure 11： Parameterization by discrete Calabi flow, generated by our software Geometric..)



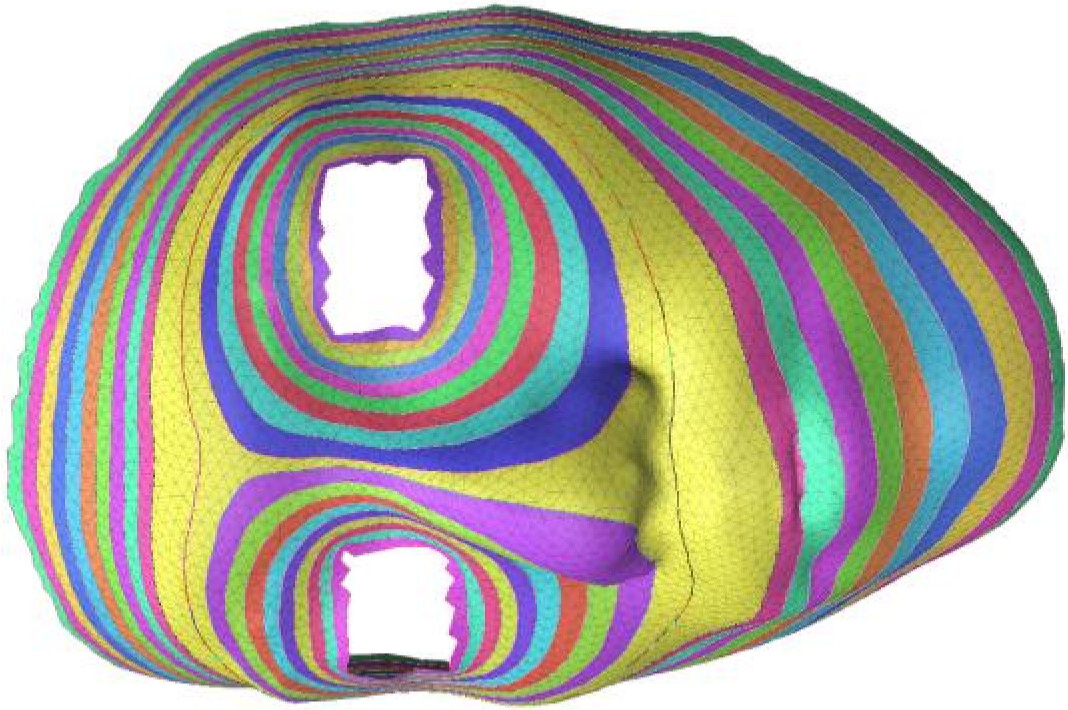
(Figure 12： A harmonic foliation with a pole, generated by our software Geometric..)



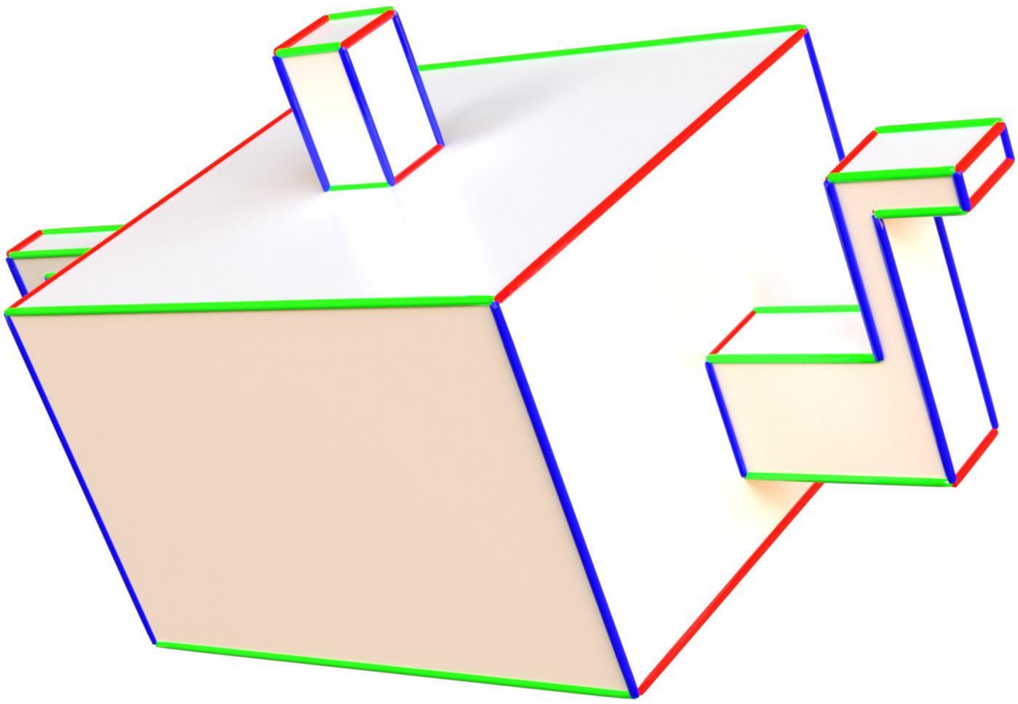
(Figure 13： A holomophic quadratic differential with a pole, generated by our software Geometric..)



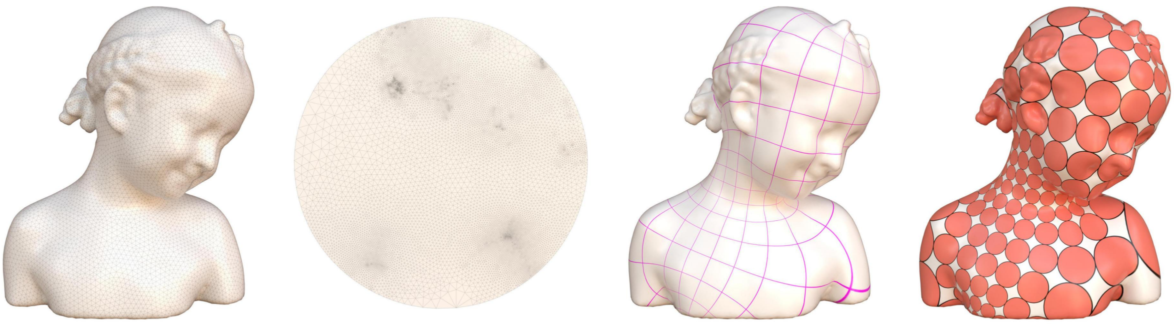
(Figure 14： A non-harmonic foliation, generated by our software Geometric..)



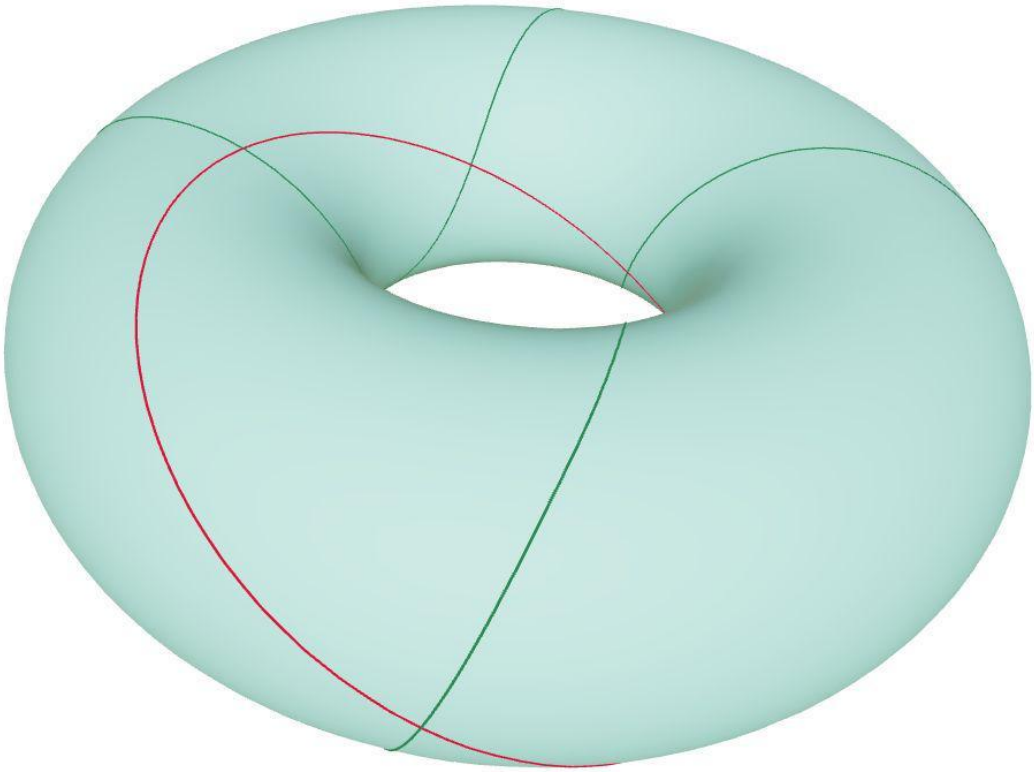
（Figure 15： A non-harmonic foliation on a surface with boudaries, generated by our software Geometric.）



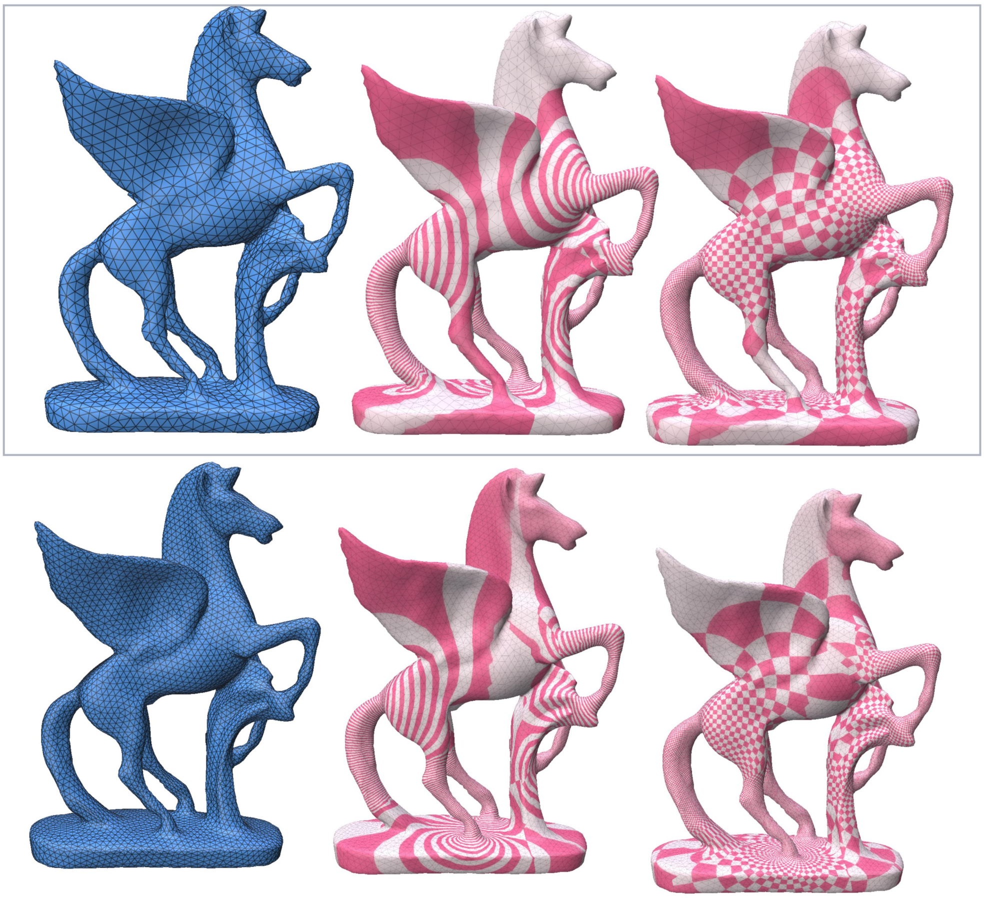
（Figure 16： An application of one form， generated by our software Geometric.）



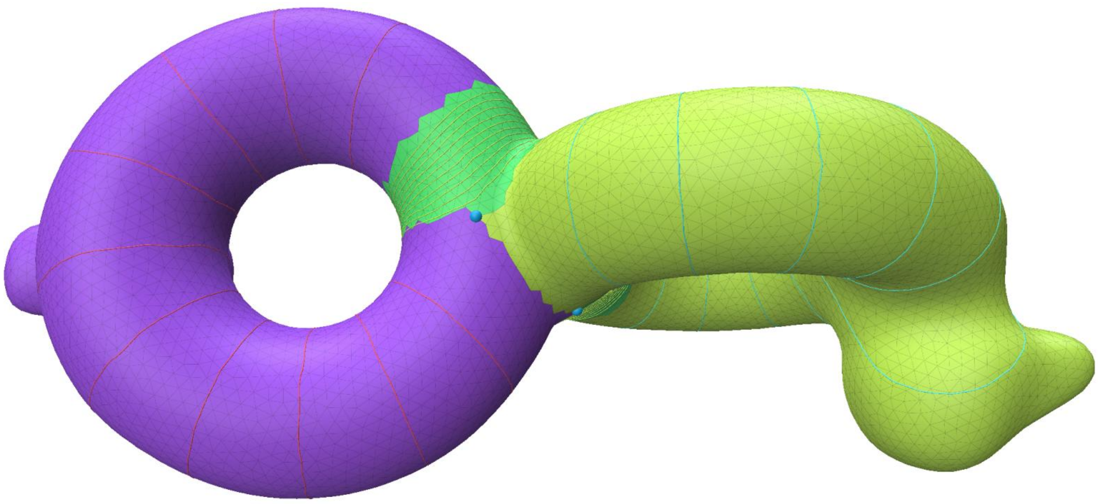
(Figure 17： Parameterization by discrete Calabi flow, generated by our software Geometric.)



(Figure 18： Several foliations and square tiling, generated by our software Geometric.)



(Figure 19： Foliations and holomophic quadratic differentials, generated by our software Geometric.)



(Figure 20： A non-harmonic foliation , generated by our software Geometric.)

3 Concepts of Geometry and Topology

Although the images exhibited in the national traveling art exhibition only cover about a dozen geometric topology concepts, each concept is linked to ten new geometric topology concepts behind it—and each of these ten concepts further derives ten related concepts. Through such hierarchical extension, one will soon encounter hundreds or even thousands of mathematical concepts rarely encountered in daily life. In fact, only by thoroughly understanding these supporting indirect concepts can one truly comprehend the dozen or so directly presented concepts at the outermost layer.

Therefore, through the partial concepts presented in this exhibition, the audience can not only learn about the concepts themselves but also gradually delve deeper to understand other related concepts behind them, such as the following:

(1) Moduli Spaces

(2) Teichmüller Spaces

(3) Dynamical Systems on Surfaces

(4) Ribbon-Graphs

(5) Harmonic Foliations

(6) Holomorphic Quadratic Differentials

(7) Meromorphic Quadratic Differentials

(8) Thurston Norm

(9) Translate Surfaces

(10) Half-Translate Surfaces

(11) Flat Surfaces

(12) Riemann Surfaces

(13) Square-Tiled Surfaces

(14) Masur-Veech Volume

(15) Meanders

(16) Billiards

(17) Interval Exchange Transformations

(18) Teichmüller Flow

(19) Strata of Abelian and Quadratic Differentials

(20) Research on surfaces and 3-manifolds by the Thurston School, etc.

...

The aforementioned geometric structures all belong to global geometric structures, which are closely related to topology and defined globally on surfaces. In contrast, there are local geometric structures such as Gaussian curvature and mean curvature, whose definitions are limited to local regions.

Currently, mathematics education and research in China are in full swing, with a large number of abstract mathematics courses and videos available online. For the concepts covered in our exhibition, as well as more geometric topology concepts and theories, references can be made to the seminars and short-term courses of the Tsinghua Yau Mathematical Sciences Center, and the academic lectures of the Peking University Beijing International Center for Mathematical Research (BICMR) in China.

4 Computational Discrete Global Geometric Structures

The core content of this national traveling art exhibition is "computational discrete Global Geometric Structures"—an original research direction we proposed in 2022.

(1) Global Geometric Structures is a concept in mathematics.

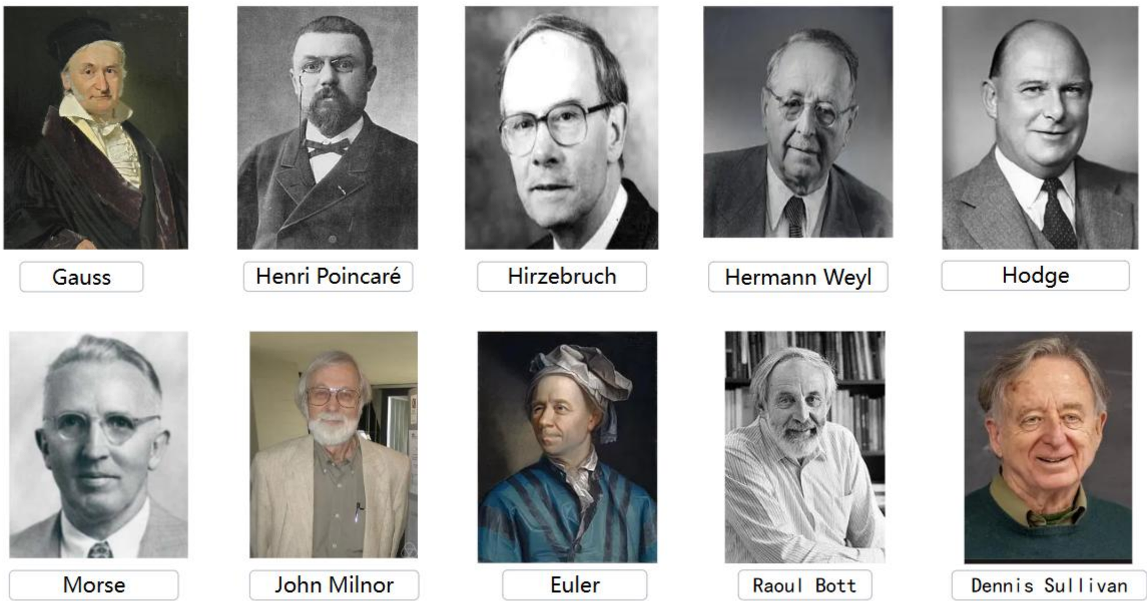
(2) Discrete refers to the discretization of the manifold or fiber bundle where global geometric structures reside.

(3) Computational means designing algorithms under the discretized framework to capture the global properties of these global geometric structures.

The art exhibition primarily showcases concepts related to global geometric structures, rather than content from computational geometry or local differential geometry — areas that audiences are already relatively familiar with.

4.1 Global Geometric Structures

Before the 1930s, local differential geometry had been relatively well-researched, and many scholars at that time believed that the research space in this field was quite limited. It was not until the 1940s that the mathematical master Chern Shiing-shen opened up a brand-new field of "global geometry" through groundbreaking works such as the proof of the Gauss-Bonnet theorem and the construction of Chern classes. The core of this field is to establish a connection between local geometric information and global topological properties, which can be regarded as a breakthrough development of the traditional local classical differential geometry.



（Figure 21： some topologists。）

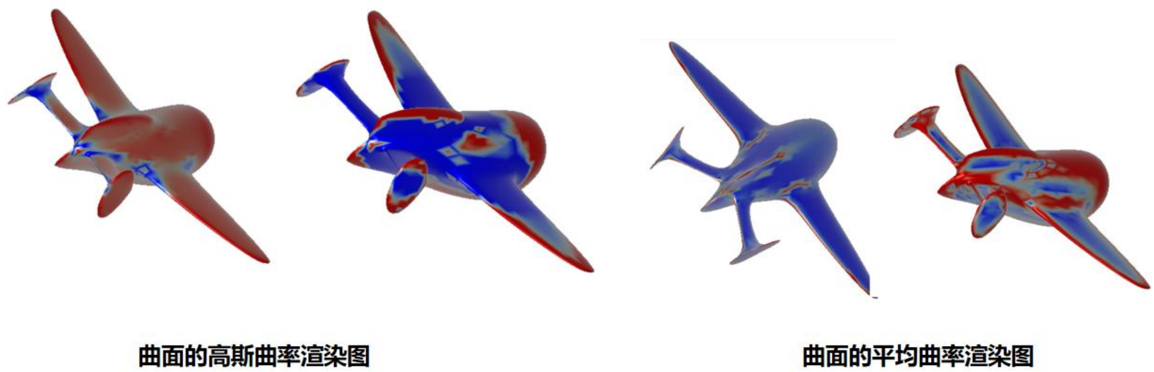
In the following decades, through the deep research of many Fields Medalists such as Shing-Tung Yau, Bill Thurston, Curtis McMullen, Maryam Mirzakhani and their colleagues in the mathematical community, the cutting-edge geometric topology theories related to global geometry gradually matured. For example, Professor Shing-Tung Yau used geometric analysis methods to prove the existence of the Calabi-Yau global geometric structure, and Thurston achieved landmark results in the fields of foliation structures and the classification of three-dimensional manifolds.

Global geometry can be regarded as a more refined branch of topology. Based on topological research, it focuses on the analysis of more precise geometric structures. Regarding the mathematicians who have made pioneering contributions to topology.

Global Geometric Structures refer to geometric structures defined on various fiber bundles over a manifold; these fiber bundles can be tangent bundles or other types of bundles. Mathematicians have discovered and developed hundreds of different global geometric structures, such as vector fields, differential 1-forms, holomorphic 1-forms, holomorphic quadratic differentials, meromorphic 1-forms, and harmonic foliations. The core research direction in this field is to determine: whether a specific global geometric structure exists on a given manifold or its fiber bundles, and what properties such a structure possesses. Taking mathematicians like Shing-Tung Yau and Gang Tian as examples, they mainly use partial differential equations (PDEs) and geometric analysis methods to study the existence of various global geometric structures, including Calabi-Yau structures.

Unlike global geometric structures, local geometric structures (e.g., Gaussian curvature, mean curvature) can be expressed through formulas or equations under a single coordinate system. In contrast, global geometric structures—such as harmonic foliations—are defined over the entire manifold. Their concepts and definitions cannot be described by a single equation; instead, they must be jointly represented by different formulas and equations under multiple coordinate systems. These equations, belonging to different coordinate systems, need to satisfy specific transformation rules, and the core information of the global structure is precisely contained in the transformation relationships between these equations.

Each type of global geometric structure has a unique equation form and set of transformation rules. Therefore, the research on global geometric structures involves developing various tools, methods, and techniques to study these transformations, thereby deriving various properties of the geometric structures that are globally related to the manifold’ s fiber bundles.



(Figure 22：Guassian curvature and mean curvature，curvature numerical value are colored，generated by meshDGP.)

A large portion of mathematicians’ work on various global geometric structures centers on proving existence—that is, verifying whether a specific type of global geometric structure exists. Even if a global geometric structure can be expressed by equations in a local coordinate system, it may not hold globally across the entire fiber bundle.

On the basis of proving existence, mathematicians then proceed to study the various properties and characteristics of the global geometric structure. Many mathematicians have made significant contributions to the research and development of global geometry, pioneering numerous new fields in the discipline.

4.2 Global Geometric Structures and Physical Constructions

Although global geometric structures possess the characteristic of globality, their definitions ultimately still rely on local equations. However, if equations are constructed arbitrarily under a certain coordinate system, they often fail to hold at the global level and thus lack research value.

An important way to derive meaningful equations is from observations of physical experiments. Conclusions from physical experiments are objective realities already established by nature, which means the corresponding equations themselves already have a foundation for validity—they have simply not yet undergone systematic mathematical research. Therefore, such global geometric structures derived from physical phenomena have actually been "verified" for their existence by nature. Even if they have not yet been proven by mathematicians using rigorous mathematical methods, they are bound to hold significant mathematical research value. Typical examples include the Dirac equation proposed by Dirac and the general relativity equations established by Einstein.

In addition to global geometric structures defined by equations originating from physics, there are also those rooted in mathematics itself, often in the form of conjectures. Examples include the Poincaré conjecture and the Calabi conjecture. These conjectures are proposed largely because they are deemed likely to be valid, and thus hold research value.

（Figure 23： some mathmaticians of global geometry。）

Physics research relies on physical experiments, and conducting experiments requires substantial external resources and conditions—such as experimental equipment—which are often difficult to satisfy easily. In contrast, for thousands of years since Euclid, mathematicians have used "abstract thinking" as their core tool, freeing themselves from the constraints of external physical conditions and enabling them to advance unburdened. This way of thinking, unshackled by reality, has driven humanity to create numerous achievements in mathematical civilization that are only limited by the "boundaries of thinking"; some of these mathematical structures may not yet have manifested in nature.

Starting from abstract concepts, mathematicians can deduce new mathematical structures based on existing ones, but it is no easy task to identify new structures that are truly valid — especially global geometric structures. If an equation is constructed out of thin air, it will most likely fail to hold globally across fiber bundles and thus naturally lack research value. Physicists, however, begin their research directly with objective results already realized by nature; therefore, any new structure discovered in physics is bound to have significant mathematical research value, as its existence has been verified through physical experiments.

Once the equations and mathematical structures derived from physical experiments are thoroughly studied by mathematicians, they can break free from the constraints of external conditions for physical experiments. For instance, some physical experiments are difficult to conduct due to enormous investment, but mathematicians can conduct comprehensive research on the relevant physical equations at almost zero cost, draw "abstract" conclusions, and in turn feed these findings back to guide physical experiments.

Furthermore, even if there were no cost constraints on physical experiments and conclusions could be obtained directly through a large number of experiments, it would still be necessary to use "abstract" mathematical tools to conduct systematic research on them. Only in this way can we ensure that all conclusions are consistent, rigorous, logical, and extensible—rather than remaining a simple accumulation of scattered results.

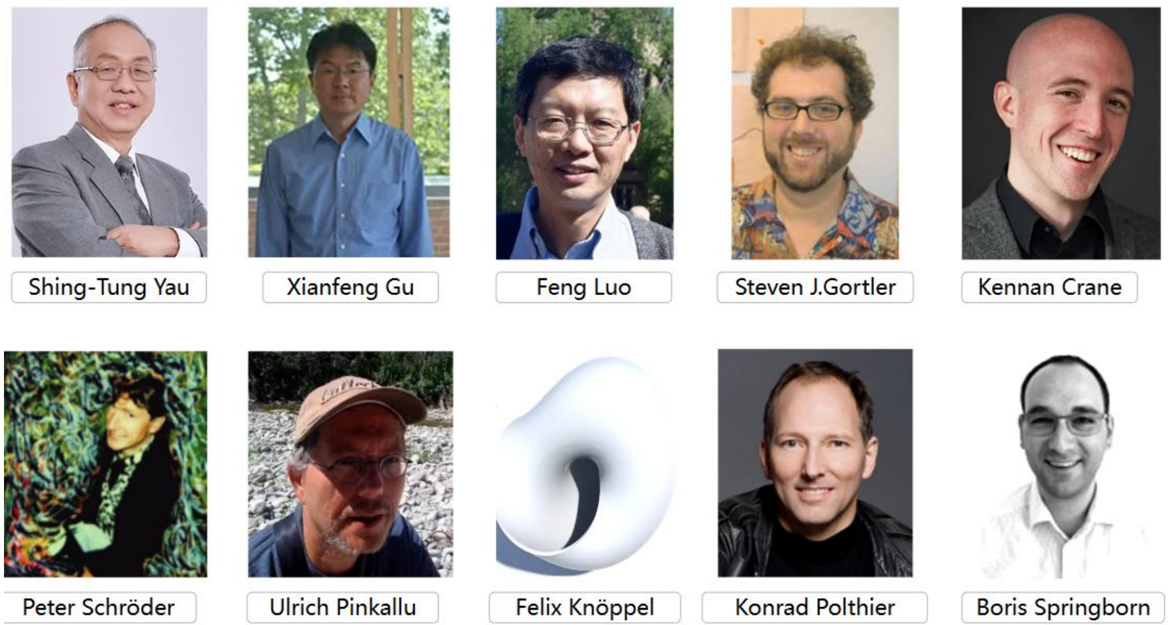
We hold the view that: "Using abstract thinking as a tool to break free from the constraints of external material conditions, and exploring new geometric structures worthy of research from the results of physical experiments already constructed by nature—this precisely reflects the inherent power of mathematics and the in-depth connection between mathematics and physics."

4.3 Global Geometric Structures and Computability

Mathematical research in the field of global geometry has gradually developed since the 1950s. Therefore, the diffusion of such mathematical theories from the mathematics community to the field of engineering technology is still in progress, and most experts in engineering technology are not yet sufficiently familiar with or proficient in these theories.

To apply the relevant theories to engineering technology, on the basis that the mathematics community has already proven the existence of these global geometric structures, the engineering community must realize the computation of these structures. Only when computational results are obtained can they be further applied to various engineering and technical scenarios.

The development of local differential geometric structures in the past followed the same path: from abstract theory to computable implementation, and then to industrial and technological application. The only difference is that the mathematical theories of local geometric structures matured earlier; after hundreds of years of development, a large number of mature computational methods have been formed.



（Figure 24： Some mathmaticians and computer scientists of computational global discrete geometric structures。）

Around the year 2000, driven by their own research needs, some computer scientists began collaborating with differential geometers and ventured into the field of computing global geometric structures. These research efforts originally emerged from the mesh research direction in computer graphics; after more than two decades of development, some global geometric structures have gradually transitioned from abstract mathematical theories to methods capable of numerical computation, and have further found applications in the industrial sector. The figure below showcases some scientists who have contributed in various ways to the computabilization of global geometric structures.

When computing global geometric structures, computer graphics scientists primarily adopt the approach of discretizing smooth surfaces into meshes, then designing algorithms on these meshes for computation. The development of this field was initially driven and motivated by specific applications. For example, to study the application of mesh parameterization and flattening, Professors Xianfeng Gu (Harvard University), Shing-Tung Yau, and Steven J. Gortler conducted research on the computable theory, algorithm design, and code implementation of differential 1-forms (a type of global geometric structure) around the 2000s (see Figure 15). Around 2010, Professor Keenan Crane from Carnegie Mellon University (CMU) carried out similar work — focusing on the computable theory, algorithm design, and code implementation of vector fields (another type of global geometric structure). Notably, the motivation behind these computability studies of global geometric structures all originated from mesh processing applications in the field of computer graphics.

In addition, other global geometric structures have also been successfully made computable by other computer scientists in the computer graphics research community, such as frame fields, spinor structures on surfaces, and conformal structures. It is worth noting that many computer scientists in this field have an educational background or experience in mathematics — particularly in differential geometry and topology — or they collaborate with differential geometers. This requires proficiency in both mesh-based algorithms and code in computer graphics, as well as the contemporary theories of differential geometry.

4.4 Propose a New Research Field

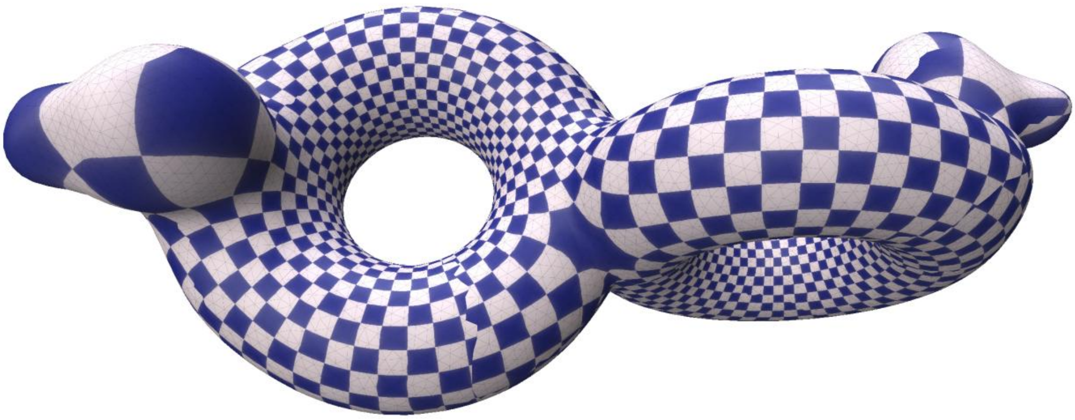
Among various global geometric structures, foliations are a notable type. The diagrams drawn by Thurston and Sullivan on the corridor walls of the Department of Mathematics at the University of California, Berkeley, have been preserved for decades, inspiring generations of teachers and students to research geometric topology. In the 1970s, Fields Medalist Thurston conducted in-depth research on foliations and achieved significant progress.

On 2D surfaces, foliations yield intuitive visual results and can be understood as a set of parallel lines. We carried out algorithmic research on foliations on 2D surfaces: building on the work of Professor Steven J. Gortler from Harvard University and others, we designed a convergent algorithm for generating harmonic foliations. Differential 1-forms can be regarded as a subset of harmonic foliations, while harmonic foliations themselves represent an extension of differential 1-forms. If a set of horizontal harmonic foliations is rotated by 90 degrees, a set of vertical harmonic foliations is obtained; combining these two sets results in another related global geometric structure: holomorphic quadratic differentials.

Based on our research insights and practical experience in the fields of harmonic foliations and super-structured quadrilateral meshes, as well as our analysis of relevant studies by other scholars, we have proposed and established a new research direction: "computational discrete Global Geometric Structures"—clearly defining its research scope, core content, and methodological system. The core of this research lies in exploring algorithm design, code implementation, and industrial applications for various "global geometric structures" developed by mathematicians, with a particular focus on those defined on 2D surfaces.

Prior to this, although algorithms for certain "global geometric structures" (such as vector fields, differential 1-forms, and frame fields) already existed, scholars mostly conducted research from different perspectives and frameworks—such as computational conformal geometry and discrete exterior calculus. Drawing on our own practice and judgments regarding the future development of the computational field of differential geometry and topology, we believe these studies can all be unified under the new framework of "computational discrete Global Geometric Structures" that we proposed. Specifically, with "the computability of global geometric structures" as the core guiding principle, this framework focuses on algorithmic research for various "global geometric structures" and systematically conducts discretization algorithm design.

The evolutionary path—Local Differential Geometry → Topology → Global Geometry → Global Geometric Structures → computational discrete Global Geometric Structures—clearly demonstrates the development trajectory of abstract mathematics gradually advancing toward industrial applications, and more closely aligns with the inherent logic of the field' s evolution.



（Figure 25： A holomophic quadratic differential， generated by Geometric。）

Currently, global geometric structures are in the historical process of evolving from abstract theories to computable implementations. The algorithmic research on computational discrete global geometric structures is not a simple calculation of abstract global geometric structures; instead, it first requires discretizing the manifold on which these structures depend, followed by algorithm design on the discretized manifold.

The core goal is to preserve the global properties of global geometric structures (as they exist on smooth manifolds) during the discretization process, rather than merely achieving local approximation. This discretization process gives rise to new geometric objects and research tools. In fact, new tools and methods developed based on the computabilization of some global geometric structures have successfully solved many long-standing challenges in industrial technology. For example, the index counting technique proposed by Professor Steven J. Gortler from Harvard University in his algorithmic research on differential 1-forms (a type of global geometric structure) was later applied to the field of rigidity, addressing numerous technical problems in this area.

Among the global geometric structures studied by the mathematics community:

(1) Some can be naturally generated through physical processes in nature;

(2) Others can be artificially constructed using computer algorithms;

(3) A large number of structures, however, remain in the stage of abstract theory—they have neither corresponding physical entities nor algorithmic constructions.

Nevertheless, with technological advancement, it is expected that the construction of these structures will be realized in the future through innovations in physical materials or breakthroughs in algorithms.

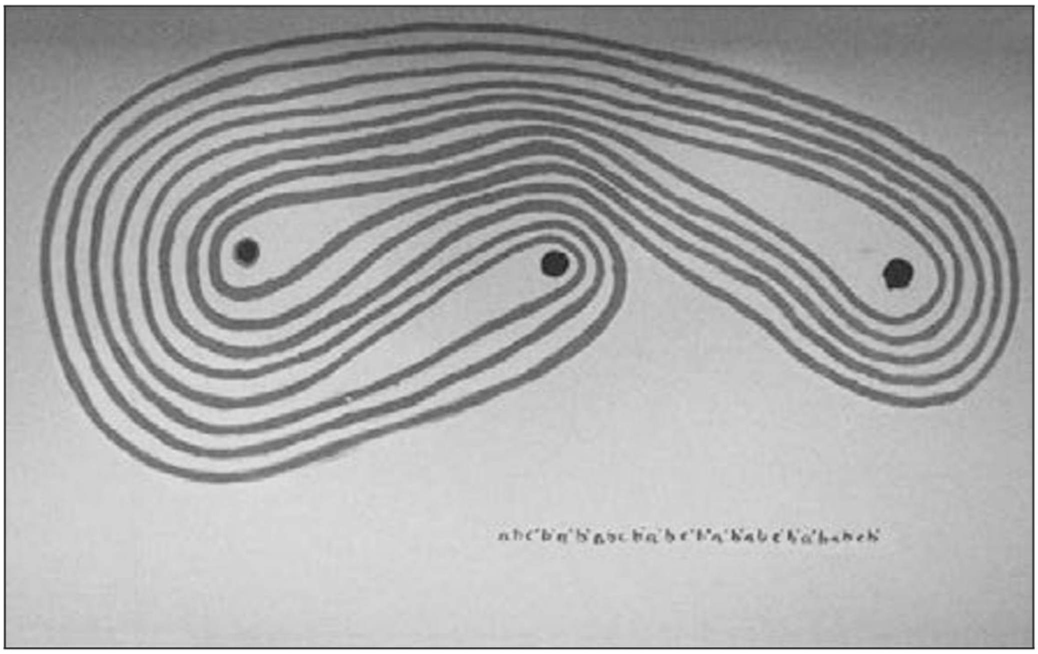
4 Visualization of Global Geometric Structures

When researching geometric topology theories, mathematicians have long created numerous hand-drawn sketches to aid understanding. A notable example is the foliation diagram drawn in the 1970s by Fields Medalist Bill Thurston (while he was a doctoral student) and Dennis Sullivan on the corridor walls of the Department of Mathematics at the University of California, Berkeley. This hand-drawn work remained on the corridor walls for decades, inspiring generations of teachers and students in their study of geometric topology theories.

However, hand-drawn sketches suffer from inherent limitations: they cannot be flexibly adjusted in terms of color, lighting, or viewing angle, and their quality heavily depends on the individual mathematician’s drawing skills.

By leveraging computer graphics rendering technology, we can achieve highly realistic visualization effects. Furthermore, through programming software, we can rotate the models and observe the renderings from a 360-degree perspective—achieving visual results that hand-drawn sketches could never match.

Although Thurston also carried out some academic popularization activities on computer-generated visualizations of geometric topology theories in the 1980s, these efforts could not be scaled up to a wider scope as they are today, due to the constraints of the technical conditions at that time.



（Figure 26：A foliation，drawed by Thurston William and Dennis Sullivan。）

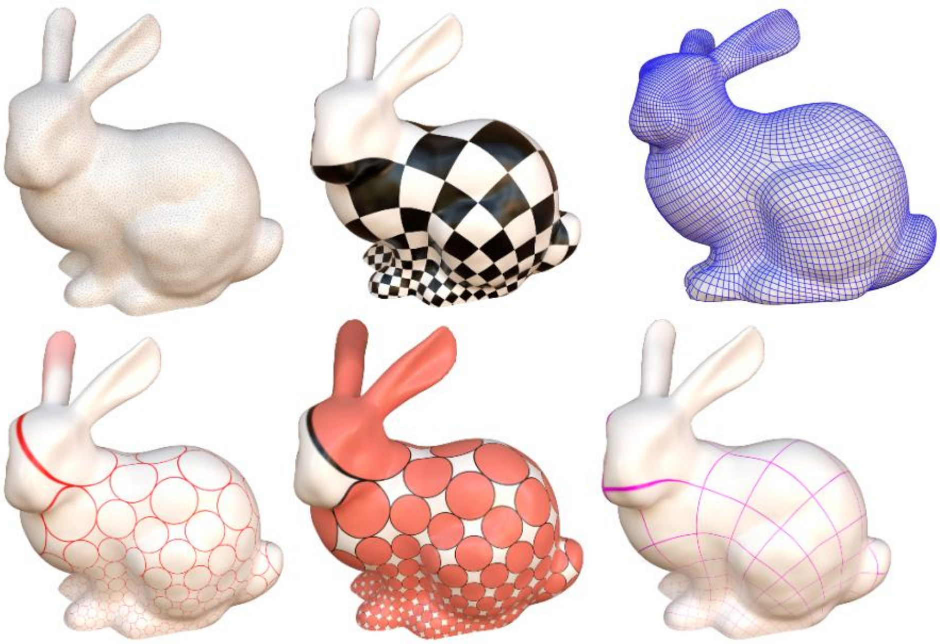
Secondly, computer graphics rendering technology is used to visualize the results of algorithmic research, rather than directly drawing "abstract" mathematical formulas. This is a common misunderstanding among researchers such as mathematicians who are not familiar with algorithm concepts and rules when creating mathematical visualizations. Concepts and theories expressed through mathematical formulas must first have their essential properties captured by algorithms, and only then can the captured results be visualized. Particularly for global geometric structures — which cannot be described by a single formula or equation — directly calculating formulas and equations fails to capture their global properties. Therefore, the main factor restricting the development of geometric topology concept visualization lies in the fact that "algorithmic" research cannot be accomplished overnight.

Typically, in engineering and technical research, emphasis is often placed on parameters such as the efficiency of algorithms or applications, with "visualization" treated merely as a byproduct. In contrast, we propose that "visualization" should be regarded as a necessary condition and essential tool for researching global geometric structures. This conclusion is derived from our own research experience: during the algorithm design process, visualizing various geometric topology elements allows us to obtain feedback through visual observation, which in turn facilitates algorithm optimization.

In the field of computability research on global geometric structures, the value of "visualization" — enabled by computer graphics rendering technology — extends far beyond the visual presentation of research results. The entire process, from algorithm conceptualization and design, to code writing and implementation, and finally to program debugging and optimization, is inseparable from visualization technology's intuitive presentation of various elements on discrete manifolds. This paper specifically emphasizes this point, aiming to urge the research community to attach greater importance to visualization technology, thereby promoting in-depth progress in the research of various computational discrete global geometric structures.

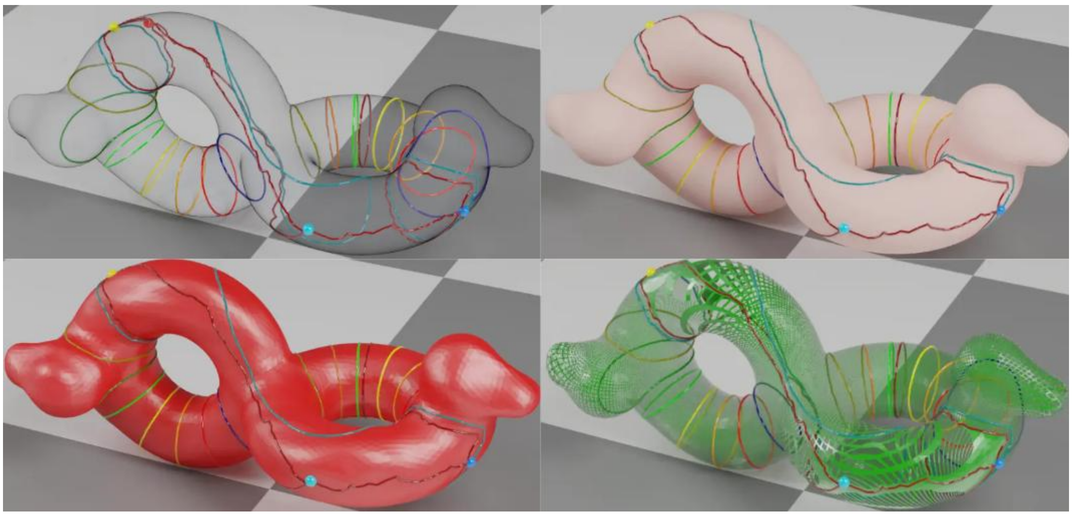
Research on the visualization of global geometric structures is of crucial importance. The exploration of algorithms for any type of global geometric structure cannot be achieved in one step; each structure requires new ideas and tools for algorithm design. Therefore, before a mature algorithm is developed, approximate "visualization" technology can be used to generate visual representations. Although such visualizations may lack global consistency and fail to satisfy the global properties of the structure, they are still based on calculation results from local formulas and equations. As a result, they can roughly reflect the characteristics and laws of the global geometric structure at the visual level, thereby providing inspiration for research.

A typical example is the visualization of holomorphic quadratic differentials: what is presented is a product of "visualization" rather than an accurate representation that strictly satisfies the global properties of holomorphic quadratic differentials. The reason is as follows: while there exists an algorithm that ensures the global properties of the component parts of holomorphic quadratic differentials (i.e., horizontal harmonic foliations), the parts obtained by rotating these foliations by 90 degrees remain approximate results. The combination of the two only forms a visually approximate presentation, not an approximate solution in the mathematical sense.



（Figure 27： Several renderings of a bunny mesh， generated by Geometric。）

The results of algorithmic research can also be presented through various rendering effects, as shown in Figures 27 and 28—effects that cannot be achieved through hand-drawn methods.



（Figure 28： Several renderings of a Ribbon graph， generated by Geometric。）

5 Algorithm vs. Computation

In the new research field of "computational discrete Global Geometric Structures" proposed by us, the core of "computability" lies in "algorithms," rather than simple "formula application" or "theory application"—that is, it is not about inputting a formula and performing numerical calculations on it.

For many local geometric structures, it is indeed feasible to perform calculations by inputting formulas and using computers for approximate estimation. However, "global geometric structures" are different: although local formulas exist for them, their global definition requires constructing relationships through transformations between multiple formulas. A single local formula or equation cannot fully define a global geometric structure. Therefore, the approach of directly calculating by inputting formulas or equations into a computer fails to yield results with "globality."

We specifically emphasize this point in a separate section for the following reason: most experts in engineering and technology are accustomed to the idea that mathematics mostly involves completing calculations through approximation methods (e.g., truncating the first few terms of a Taylor expansion and using a computer to obtain results). In other words, they are more familiar with the mindset of "inputting a formula or equation, then calculating directly via manual computation or software," and have less exposure to calculation methods centered on "algorithms." Additionally, most technologies in this field involve local geometry, where results can be obtained through such "calculation" methods—this has further solidified the perception of experts in related fields. However, technologies that can be solved solely by relying on local calculations are already relatively mature.

This perception gap is particularly prominent in practice: many experts in this area consider the research focus to be on "formulas and equations," with subsequent calculations being mere afterthoughts. In contrast, our view is that the key to research lies in "algorithms," and the preceding formulas and equations all serve the algorithm. In short, we need to design "algorithms" to capture the global properties of global geometric structures, rather than relying on local formulas within global geometric structures for calculations. The latter will lead to the loss of global properties due to calculation errors, ultimately failing to truly obtain the global geometric structure—and thus providing no help in solving engineering and technical problems using global geometric structures, as we propose.

Currently, the inherent perception of relying on local calculations is relatively widespread, stemming from the fact that the application of global geometric structures in industrial fields has been relatively limited in the past. As the research field of "computational discrete Global Geometric Structures" we proposed becomes more widespread, the essential distinction between "algorithms" and "calculations" will undoubtedly gain broader recognition.

6 The Beauty of Global Geometric Structures vs. The Beauty of Mathematics

To popularize mathematical knowledge, many scholars and experts have delivered lectures and presentations on topics such as the beauty of mathematics, geometric art, mathematics and beauty, and science and art. The goal is to use art and beauty—concepts that everyone can understand and perceive—to help audiences indirectly gain a preliminary understanding and sense of abstract, hard-to-comprehend mathematical concepts.

These presentations' interpretations of "mathematics and beauty" generally fall into three categories:

(1) Perceiving beauty "spiritually and mentally" only after understanding profound mathematical and geometric theories

The logic of such presentations is to inspire audiences to learn abstract mathematics, with the promise that they will be able to perceive beauty once they have mastered the subject. However, they fail to address a key challenge: how to let audiences first develop an intuitive visual perception of beauty — before understanding the relevant mathematical concepts—and then be drawn to learn those concepts afterward.

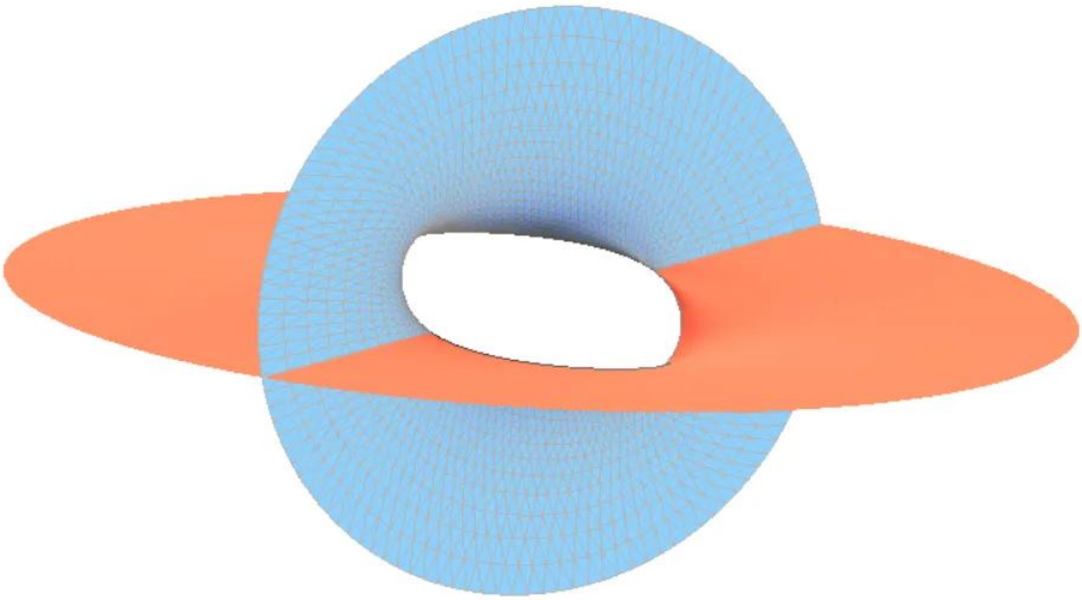
(2) Perceiving visual beauty through the external "shapes" of certain geometries placed in 3D space

Many mathematical concepts in local geometry have intuitive shapes that can be visually observed, such as the Möbius strip. Audiences can learn about these geometric concepts by viewing sculptures, images, or other works that directly correspond to such shapes.

Because this type of "mathematical beauty" is accessible through intuitive shapes, it has become quite common in popular science. Many mathematics departments now have related sculptures: for example, the Umbilic Torus sculpture at the Department of Mathematics of Stony Brook University (New York), and the Calabi-Yau sculpture in the square in front of the Shanghai Institute for Mathematics and Interdisciplinary Sciences.

(3) Perceiving mathematical beauty through natural phenomena that conform to mathematical theorems

A third category involves natural phenomena — such as plant growth — that follow specific mathematical theorems. Audiences perceive mathematical beauty through observing these phenomena, as they witness how abstract mathematical rules manifest in the real world.

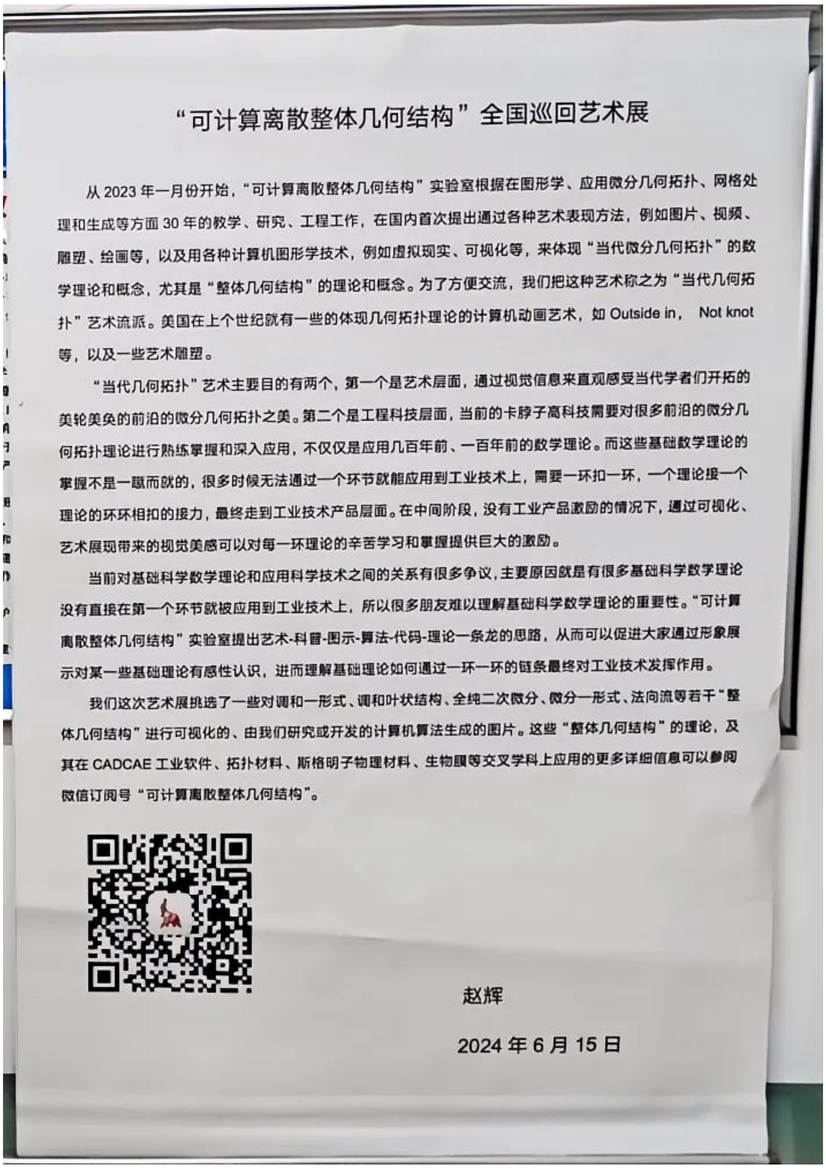


(Figure 29：Calabi-yau picture，generated by meshDGP。)

However, the geometric beauty presented in our national touring art exhibition differs from the three categories mentioned above. What makes our exhibition unique is that it does not focus on geometric "shapes" in 3D space, but rather on intrinsic "geometric structures" on surfaces — such as harmonic foliations, ribbon graphs, differential 1-forms, and Calabi flows. These are not natural shapes and have no inherent visual appearance; they can only be captured through computer algorithms and then mapped into visually perceivable effects.

The transformation of the beauty of intrinsic global geometric structures into visual form via computer graphics technology represents an innovation at the intersection of computer graphics, algorithm design, and geometric topology theory. This approach can attract more teachers and students to learn about and research these intrinsic cutting-edge geometric topology theories, as well as their applications in industrial technology—thereby enabling art to serve as a "powerful tool for promoting the research and application of geometric topology theories."

To our knowledge, this type of touring exhibition featuring "intrinsic global geometric structures" is the first of its kind, both domestically and internationally.



7 Participating universities and research institutions

The research direction of computational discrete Global Geometric Structures is not limited to studies in mathematics; it mainly targets research on applied geometry in computer science and other science and engineering fields. However, frontiers of differential geometric topology theories related to "global geometric structures" are rarely accessible to teachers and students of non-mathematics majors. Talents who understand and master these theories while being able to apply them in algorithm design are extremely scarce.

Although the geometric topology theories developed by mathematicians over the past decades have been gradually penetrating into engineering fields and applied in research practices, this interdisciplinary integration process is still in a stage of continuous development. Nevertheless, due to the high abstraction of these theories, teachers and students of non-mathematics majors often need a strong learning motivation to conduct in-depth exploration. The art form, however, can effectively stimulate the interest in learning and research among this group.

Based on the author's achievements in algorithm design in the application field of geometric topology theories, and with the help of rendering technology in computer graphics, these abstract geometric topology concepts and theories have been transformed into visual presentations. On this basis, in 2024, the author launched the Global Touring Art Exhibition of computational discrete Global Geometric Structures, which has so far been held at more than ten universities and research institutions across the country, including:

(1) Zhongyuan University of Technology,

(2) Changchun Institute of Optics, Fine Mechanics and Physics, Chinese Academy of Sciences,

(3) North China Electric Power University,

(4) Henan University,

(5)Xihua University,

(6) Renmin University of China,

(7) South-Central Minzu University,

(8) Tianjin Chengjian University,

(9)Taiyuan University of Technology,

(10)High Magnetic Field Science Center, Hefei Institutes of Physical Science, Chinese Academy of Sciences,

(11) Jiangxi University of Science and Technology,

(12) Dali Li Zhengdao Science and Art Center.

The national touring art exhibition is not yet concluded, and dozens of more universities will host the exhibition in the follow-up.

The images exhibited cover geometric topology concepts such as Ribbon-graph, Square-Tiling, harmonic foliations, holomorphic quadratic differentials, meromorphic quadratic differentials, differential 1-forms, discrete Calabi flow, and normal flow.

During the exhibition, teachers and students from many universities feedback that this art exhibition has pioneered a new paradigm of "intrinsic geometric structure art" and serves as an innovative bridge connecting cutting-edge mathematical theories with public aesthetics. For the first time, the exhibition has systematically presented the intrinsic structures of surfaces that are difficult to perceive intuitively, transforming abstract theories such as holomorphic differentials and harmonic foliations into digital paintings with vivid colors, thus turning them from theoretical symbols into tangible visual entities.

Through this exhibition, the audience not only developed a strong sense of curiosity and desire for exploration but also truly felt the infinite charm generated by the interdisciplinary integration of mathematics and computer science. After visiting the exhibition, many teachers and students went to the library to look up relevant materials and learn about the mathematical concepts illustrated in the exhibition. Unconsciously, they developed an interest in these cutting-edge geometric topology theories.

Following the art exhibition, institutions including the School of Science of Tianjin Chengjian University and theSchool of Science of Jiangxi University of Science and Technology were still eager for more; they made full sets of reproductions of all over 50 artworks in the exhibition for collection and have launched long-term exhibitions on their campuses.

Compared with common exhibitions themed "Beauty of Mathematics" or "Geometric Art", this National Touring Art Exhibition has significant differences in both the core geometric and mathematical theories it presents and the external forms of presentation. Its core content lies in the "global geometric structures" generated through computer graphics technology and algorithms, and these geometric artworks are closely associated with applications in "chokepoint" fields such as industrial software, with the potential to solve relevant technical problems—which is exactly the unique value and innovative feature that previous lectures and exhibitions themed "Beauty of Mathematics" lack.

Although American mathematicians such as Thurston have popularized cutting-edge geometric theories and concepts through art forms, they were limited by the rendering technology available at that time and could hardly achieve the visual aesthetics supported by modern computer graphics. In addition, the fact that this exhibition is held as a national tour with the spontaneous participation of university teachers is an innovative attempt in itself.

Many teachers and students who came into contact with these differential geometric topology theories for the first time have summarized a large number of accurate insights and views on global geometric structures and computational research based on their own understanding, which has achieved the purpose of holding the exhibition.

Some teachers engaged in the research of these geometric topology theories also stated that the visual presentation is very helpful for their research work, as it can bring new perspectives and insights.

A number of schools have published graphic reports on the exhibition, including:

(1) Henan University,
(2) Renmin University of China,
(3) Tianjin Chengjian University,
(4) Taiyuan University of Technology,
(5) High Magnetic Field Science Center, Hefei Institutes of Physical Science, Chinese Academy of Sciences,
(6) Jiangxi University of Science and Technology.

Some schools have integrated the exhibition with other mathematical activities, achieving even better results. For example:

The 8th exchange and discussion activity of "RUC Mathematics Time I" was held;

The 8th Session of "RUC Mathematics Time I" — Geometric Structures: Theory, Algorithms and Visualization.

Another example of integration with topological materials:

The first science-art integrated art exhibition of Dali Li Zhengdao Science and Art Center opened at Taibao Home Dali Community.

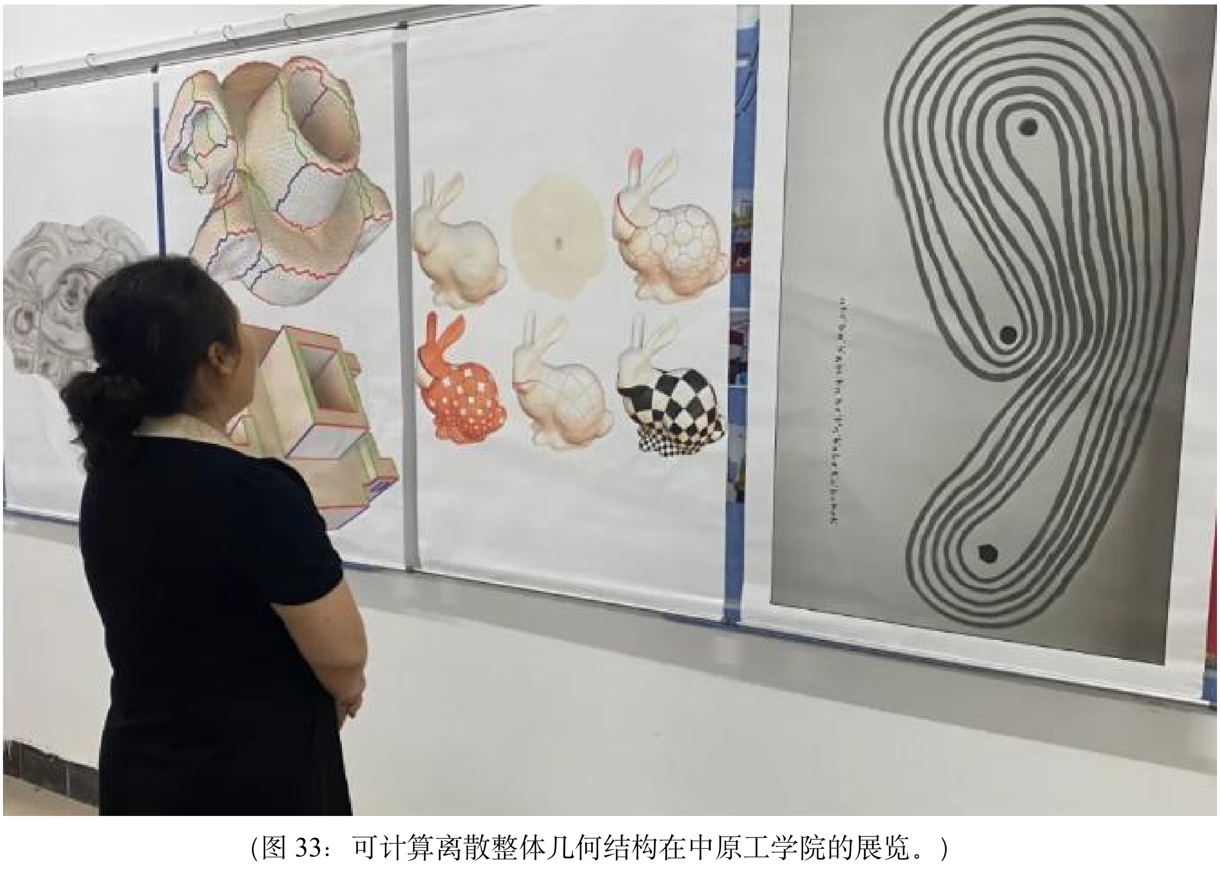
7.1 中原工学院

可计算离散整体几何结构全国巡回艺术展第一站：由中原工学院数学与信息科学学院周瑞芳副教授举办。
地点在：郑州中原工学院 2 号实验楼 005 房间（数学与信息科学学院工会小家）。
日期是：2024 年 6 月 20 日开始。

（图 30：可计算离散整体几何结构在中原工学院的展览。）

（图 31：可计算离散整体几何结构在中原工学院的展览。）

（图 32：可计算离散整体几何结构在中原工学院的展览。）



（图 33：可计算离散整体几何结构在中原工学院的展览。）

7.2 中国科学院长春光机所

可计算离散整体几何结构全国巡回艺术展第二站：由中国科学院长春光机所研发中心刘震宇教授举办。
地点在：中国科学院长春光机所研发中心孵化器 A 座大厅（长春营口路 77 号）。
日期是：2024 年 8 月 23 日开始。

（图 34：可计算离散整体几何结构在中科院长春光机所的展览。）

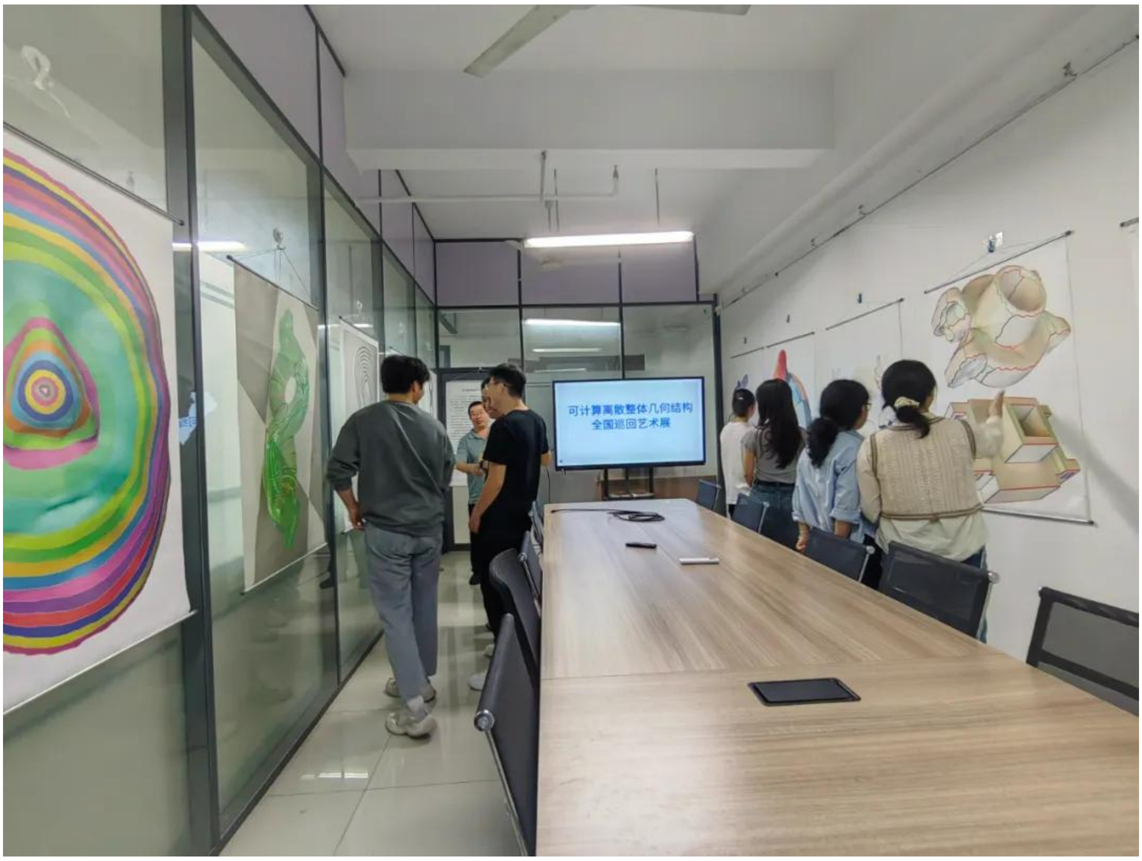
（图 35：可计算离散整体几何结构在中科院长春光机所的展览。）

（图 36：可计算离散整体几何结构在中科院长春光机所的展览。）

（图 37：可计算离散整体几何结构在中科院长春光机所的展览。）

7.3 华北电力大学

可计算离散整体几何结构全国巡回艺术展第三站：由华北电力大学新能源学院王孝强副教授举办。
地点在：北京昌平区北农路 2 号华北电力大学主楼 B 座 203A。
日期是： 2024 年 09 月 02 日开始。



（图 38：可计算离散整体几何结构在华北电力大学的展览。）

（图 39：可计算离散整体几何结构在华北电力大学的展览。）

（图 40：可计算离散整体几何结构在华北电力大学的展览。）

（图 41：可计算离散整体几何结构在华北电力大学的展览。）

7.4 河南大学

可计算离散整体几何结构全国巡回艺术展第四站：由河南大学数学与统计学院韩喆副教授举办。
地点在：开封市河南大学金明校区数学与统计学院一楼。
日期是：2024 年 09 月 19 日开始。

（图 42：可计算离散整体几何结构在河南大学的展览。）

（图 43：可计算离散整体几何结构在河南大学的展览。）

（图 44：可计算离散整体几何结构在河南大学的展览。）

（图 45：可计算离散整体几何结构在河南大学的展览。）

7.5 西华大学

可计算离散整体几何结构全国巡回艺术展第五站：由西华大学机械工程学院陈宏副教授举办。
地点在：成都市西华大学郫都校区机械工程学院（5 教 A 区）一楼。
日期是：2024 年 10 月 21 日开始。

（图 46：可计算离散整体几何结构在西华大学的展览。）

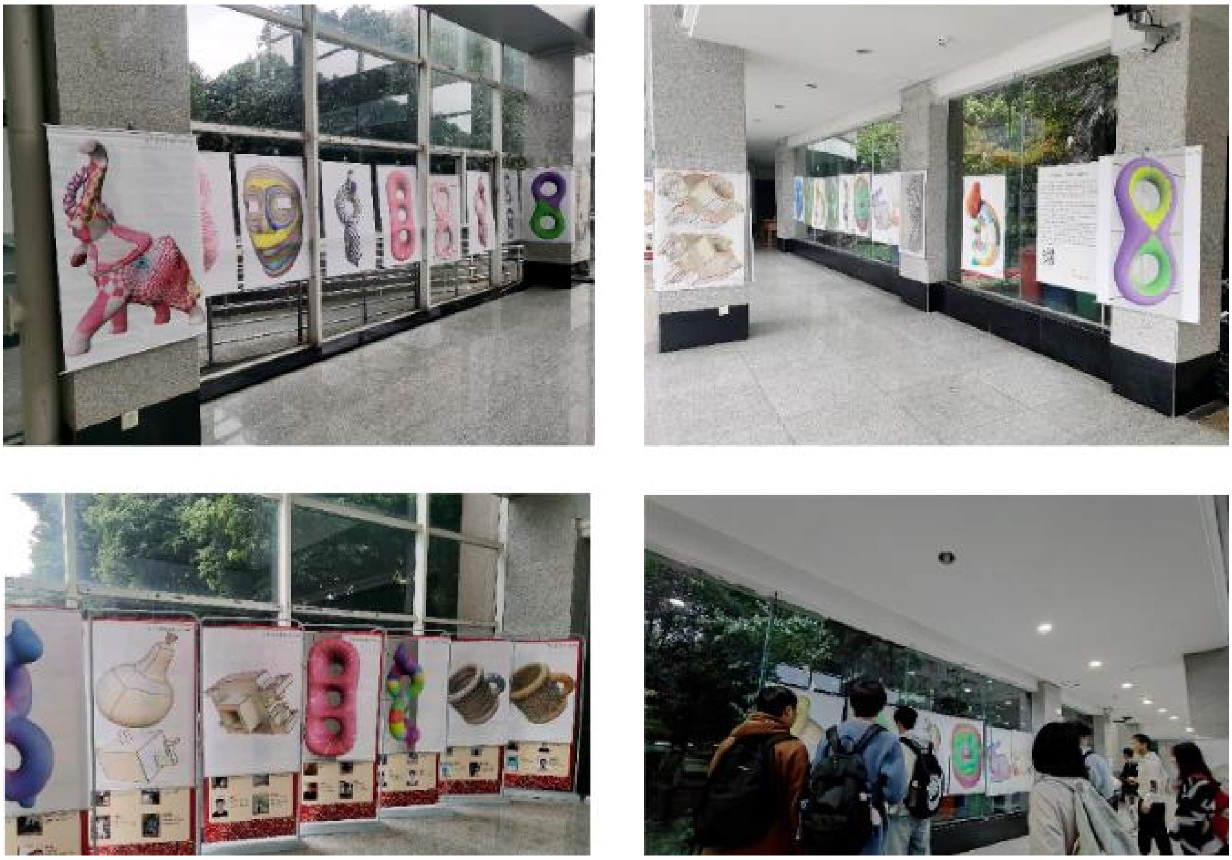
（图 47：可计算离散整体几何结构在西华大学的展览。）

（图 48：可计算离散整体几何结构在西华大学的展览。）

（图 49：可计算离散整体几何结构在西华大学的展览。）

7.6 中国人民大学

可计算离散整体几何结构全国巡回艺术展第六站：由中国人民大学数学学院葛化彬教授举办。
地点在：北京中国人民大学明德楼和图书馆大厅。
日期是：2024 年 12 月 13 日开始。



（图 50：可计算离散整体几何结构在中国人民大学的展览。）

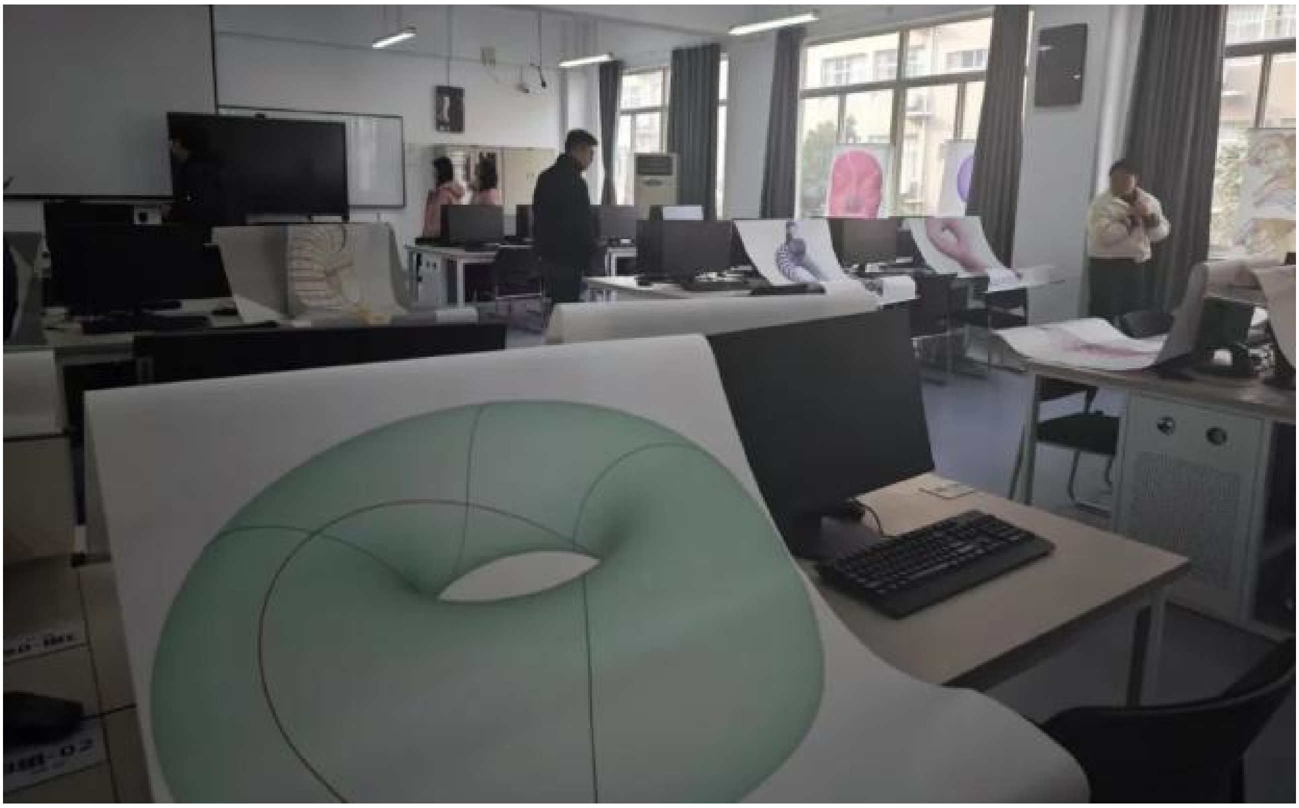
（图 51：可计算离散整体几何结构在中国人民大学的展览。）

（图 52：可计算离散整体几何结构在中国人民大学的展览。）

（图 53：可计算离散整体几何结构在中国人民大学的展览。）

7.7 中南民族大学

可计算离散整体几何结构全国巡回艺术展第七站: 由中南民族大学计算机科学学院朱剑林老师举办。
地点在：中南民族大学计算机科学学院 9 号楼 311 室。
日期是：2025 年 2 月 21 日开始。



（图 54：可计算离散整体几何结构在中南民族大学的展览。）

（图 55：可计算离散整体几何结构在中南民族大学的展览。）

（图 56：可计算离散整体几何结构在中南民族大学的展览。）

（图 57：可计算离散整体几何结构在中南民族大学的展览。）

7.8 天津城建大学

可计算离散整体几何结构全国巡回艺术展第八站：由天津城建大学理学院张东、王晓玲、王丽霞老师举办。
地点在：天津城建大学行健楼一楼大厅。
日期是：2025 年 3 月 26 日开始。

（图 58：可计算离散整体几何结构在天津城建大学的展览。）

（图 59：可计算离散整体几何结构在天津城建大学的展览。）

（图 60：可计算离散整体几何结构在天津城建大学的展览。）

（图 61：可计算离散整体几何结构在天津城建大学的展览。）

（图 62：可计算离散整体几何结构在天津城建大学的展览。）

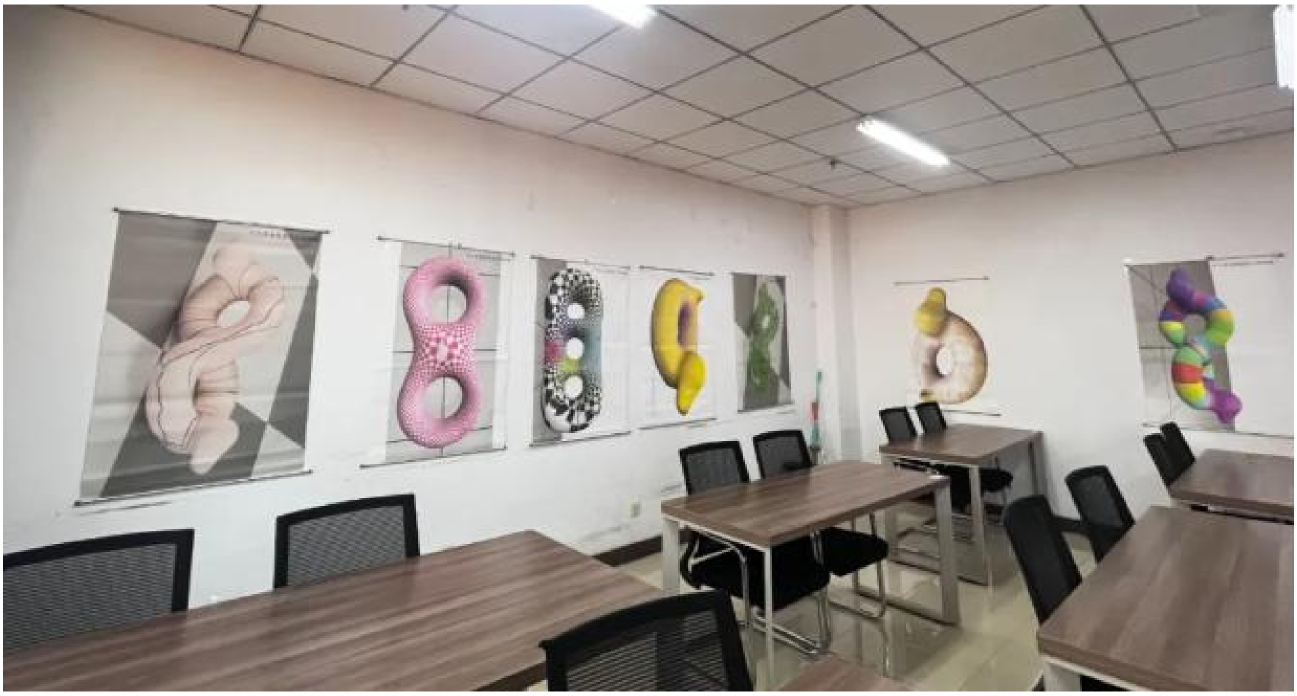
（图 63：可计算离散整体几何结构在天津城建大学的展览。）

7.9 太原理工大学

可计算离散整体几何结构全国巡回艺术展第九站：由太原理工大学雷敏老师举办。
地点在：太原理工大学明向校区数学学院 713。
日期是：2025 年 5 月 10 日开始。

（图 64：可计算离散整体几何结构在太原理工大学的展览。）

（图 65：可计算离散整体几何结构在太原理工大学的展览。）

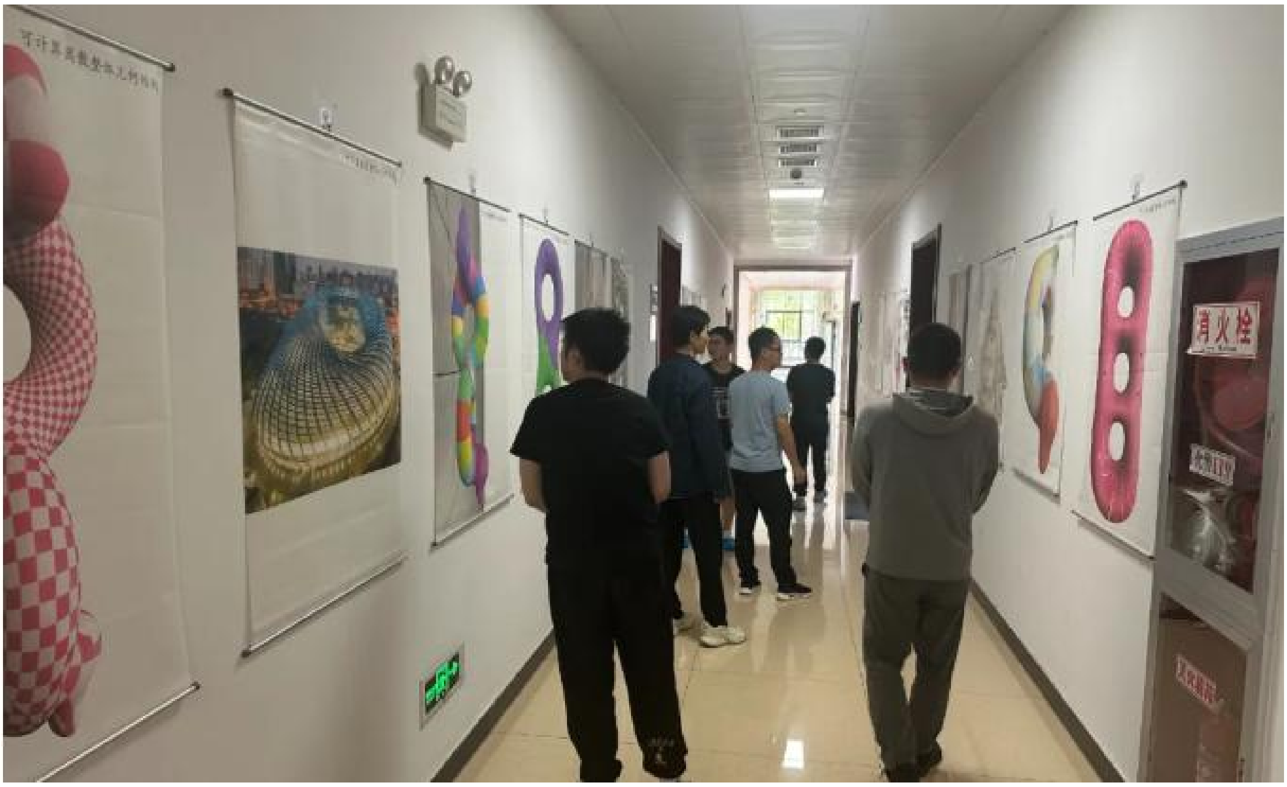


（图 66：可计算离散整体几何结构在太原理工大学的展览。）

（图 67：可计算离散整体几何结构在太原理工大学的展览。）

7.10 中国科学院合肥物质科学研究院

可计算离散整体几何结构全国巡回艺术展第十站：由中国科学院合肥物质科学研究院强磁场科学中心杜海峰教授举办。
地点在：合肥蜀山区交叉创新大楼南楼一层。
日期是：2025 年 7 月 12 日 开始。



（图 68：可计算离散整体几何结构在中国科学院合肥物质科学研究院强磁场科学中心的展览。）

（图 69：可计算离散整体几何结构在中国科学院合肥物质科学研究院强磁场科学中心的展览。）

（图 70：可计算离散整体几何结构在中国科学院合肥物质科学研究院强磁场科学中心的展览。）

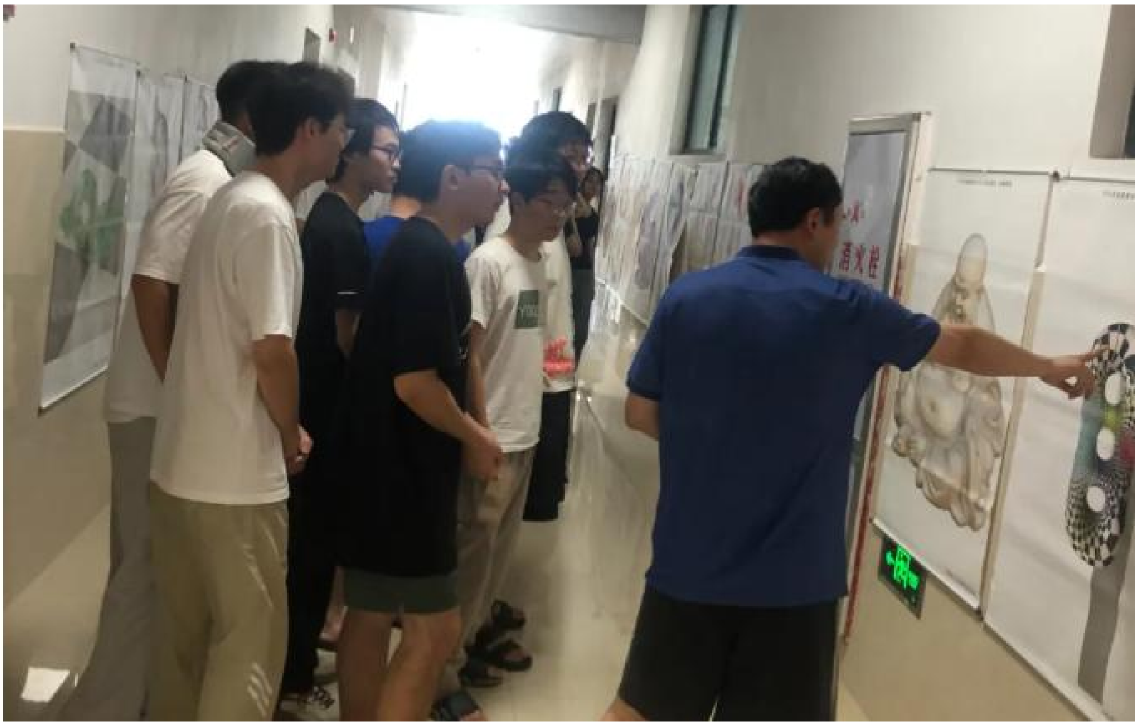
（图 71：可计算离散整体几何结构在中国科学院合肥物质科学研究院强磁场科学中心的展览。）

7.11 江西理工大学

可计算离散整体几何结构全国巡回艺术展第十一站：由江西理工大学理学院杨火根教授举办。
地点在：江西理工大学（三江校区）理学院。
日期是：2025 年 7 月 13 日开始。

（图 72：可计算离散整体几何结构在江西理工大学的展览。）

（图 73：可计算离散整体几何结构在江西理工大学的展览。）



（图 74：可计算离散整体几何结构在江西理工大学的展览。）

（图 75：可计算离散整体几何结构在江西理工大学的展览。）

7.12 大理李政道科学艺术中心

可计算离散整体几何结构全国巡回艺术展第十二站:
此次艺术展是由中国科学院院士、上海交通大学李政道研究所副所长、凝聚态物理研究部主任、2025年未来科学大奖“物质科学奖”获得者丁洪院士在大理李政道科学艺术中心举办。

地点在：云南省大理白族自治州大理市大理镇苍山国家地质公园东北 285 米太保家园·大理国际乐养社区共享大厅 2F、会议会务中心 1F。

日期是：2025 年 8 月 11 日开始。

通过大理李政道科学艺术中心展览的“整体几何结构”艺术图片，赵辉老师提出了一些全新的整体几何结构、传统计算机上的算法、量子力学（Quantum Mechanics）、拓扑材料、量子计算机之间关系的创新观点：物理上的量子，例如光子等也是一种整体几何结构，但是作用于整个宇宙，无法从外部观察，很多反直觉的量子现象都是来源于光子的整体属性。而调和叶状结构是位于二维曲面上的整体几何结构，可以通过观察调和叶状结构的整体属性，来类比光子等量子现象的整体属性。物理宇宙就类是一个量子计算机，可以以线性时间进行构造量子现象，而调和叶状结构等在传统计算机上的算法，需要以非线性的时间为代价进行构造。

（图 76：可计算离散整体几何结构在大理李政道科学艺术中心的展览。）

(1) 光子、电子等物理粒子的量子力学在数学上就是某种“整体几何结构”。
(2) 但是它们都是在整个四维 space -time 空间，所以无法从外部观测。
(3) 反直觉的量子力学现象都是这些物理粒子的“整体几何结构”里面的整体属性无法从外部观测的体现。
(4) 几何拓扑理论里还有其他的很多的抽象“整体几何结构”。
(5) 其中一些在二维曲面上，例如“调和叶状结构”，可以从曲面外部进行观测。
(6) 可以设计算法用传统计算机算出来“调和叶状结构”。
(7) 从而可以通过二维的调和叶状结构 类比 四维 space-time 的物理 Quantum Mechanics 的现象。
(8) 这是用传统的计算机 non-linear time 的算法，来类比量子计算机的 linear time 现象。
(9) 物理学家研究的马约拉纳粒子，也和一般的发生在四维的 space-time 物理粒子不一样，马约拉纳粒子也是在三维材料中。
(10) 在此次艺术展中，通过可计算的调和叶状结构和计算机图形学的渲染技术得到的视频和图片，可以生动的展示调和叶状结构里面类比的量子力学的各种现象。
(11) 创新的通过计算机算法为桥梁跨学科的联系起来了 Computer Science + Differential Geometry + Quantum+ Topologica Material。
(12) 微分几何的很多数学家，例如丘成桐教授、田刚教授等就是用几何分析方法研究 calabi-yau 整体几何结构。
(13) 但是数学家的方法是从抽象到抽象， 跨越一步到物理材料应用上会有很多实际的困难。通过“算法”的模式在中间搭桥，物理材料和抽象数学就有希望在可行的时间精力条件下打通。因为到了算法这一步，都是步骤，理工科的师生就都能很快理顺，但是从抽象数学开始，基本上很难凑够时间精力走到能应用的阶段。
(14) 量子计算机的构建不一定要依赖四维 space-time 上的物理粒子， 因为四维空间的粒子的整体几何结构是全宇宙，量子纠缠（quantum entanglement）会很容易消失。
(15) 可以采用新的不是以整体宇宙为舞台的量子现象，例如马约拉纳粒子，拓扑电子材料等可能具有量子纠缠现象的材料来构造量子计算机。根据可能生成的稳定量子材料的性质，未来的量子计算机也不一定是二进制 qubit 的，也可以是三进制、五进制、19 进制。

（图 77：可计算离散整体几何结构在大理李政道科学艺术中心的展览。）

（图 78：可计算离散整体几何结构在大理李政道科学艺术中心的展览。）

（图 79：可计算离散整体几何结构在大理李政道科学艺术中心的展览。）

8 长期收藏展览

部分高校在承办全国巡回艺术展后，校内师生仍觉意犹未尽。为进一步深化本校师生对相关几何拓扑理论及应用的理解与学习，一些院校的领导决定长期收藏可计算离散整体几何结构实验室的整套整体几何结构艺术图片，将其应用于招生宣传、工会活动、党支部、党日活动、学生会活动、科研、教育、培训等校院各项工作中。

此举开创了国内国际前沿几何拓扑理论及其应用在教学、科研、科普、社会服务、以及跨学科交叉学科融合领域工作模式的先河，展现了当代中国理工科学者与教师突破科技发展历史局限、奋勇前行的精神风貌。

8.1 天津城建大学理学院

天津城建大学理学院在 2025 年上半年进行了第八站可计算离散整体几何结构巡回艺术展之后，为了进一步深化理学院师生的了解和学习相关的几何拓扑理论和应用，经过理学院领导详细研究，院务会决定对于《可计算离散整体几何结构》实验室赵辉老师原创的 50 多副各种整体几何结构图片进行官方收藏，并长期在理学院展览，以及用于招生宣传、工会活动、党支部的党日活动等学校学院的各项工作中。

（图 80： 天津城建大学理学院招生宣传展览。）

（图 81： 天津城建大学理学院招生宣传展览。）

（图 82： 天津城建大学理学院长期收藏展览。）

（图 83： 天津城建大学理学院党支部活动。）

（图 84： 天津城建大学理学院招生宣传展览。）

8.2 江西理工大学理学院

江西理工大学理学院经过理学院领导详细研究，院务会决定对于《可计算离散整体几何结构》实验室赵辉老师原创的 50 多副各种整体几何结构图片进行官方收藏，并长期在理学院展览，以及用于招生宣传、工会活动、党支部的党日活动等学校学院的各项工作中。

（图 85： 江西理工大学理学院收藏。）

（图 86： 江西理工大学理学院收藏。）

（图 87： 江西理工大学理学院收藏。）

9　艺术展新闻发布会和研讨会

在全国巡回艺术展展览期间，太原理工大学理学院同步举办了 “几何与艺术” 线上线下新闻发布会和几何与艺术的研讨会，邀请全国十余所高校不同专业的教师作报告。

为推动可计算离散整体几何结构、CADCAE 工业软件、网格技术、图形学、前沿几何拓扑理论应用、以及相关学科的交叉融合，促进相关领域学者与行业专家的交流合作，由太原理工大学数学学院举办的“可计算离散整体几何结构艺术展发布会”于 2025 年 6 月 2 日在太原理工大学明向校区顺利举行。6 月 2 日下午，太原理工大学数学学院贺衎副院长主持发布会开幕式并致欢迎辞。

会上，山西省发展和改革委员会二级巡视员武东升教授、中国科学院孙志斌教授、哈尔滨工业大学刘绍辉教授、江西理工大学杨火根教授、天津城建大学理学院张东老师、中原工学院周瑞芳老师、中南民族大学计算机科学学院朱剑林老师、河南大学数学与统计学院韩喆老师、西华大学机械学院陈宏老师、安徽师范大学陈鹏老师、三体电视剧视觉导演陆贝珂等学者围绕几何与艺术、科学与艺术、内蕴几何与艺术等主题，分享了富有洞见的精彩观点。

（图 88：　太原理工大学理学院新闻发布会。）

（图 89：　太原理工大学理学院新闻发布会。）

（图 90： 太原理工大学理学院新闻发布会。）

（图 91： 太原理工大学理学院新闻发布会。）

可计算离散整体几何结构全国巡回艺术展第九站：太原理工大学 新闻发布会

主持人	时间	内容	发言人	单位
嘉宾	16:00-16:10	领导讲话	武东升	山西发改委
	16:10-16:15	院领导发言	贺衎	太原理工大学数学学院
	16:15-16:30	太原理工大学全国巡回艺术展介绍	雷敏	太原理工大学数学学院
嘉宾	16:30-16:35	全国巡回艺术展第八站 心得、体会、收获、经验	张东	天津城建大学理学院
嘉宾	16:35-16:40	全国巡回艺术展第七站 心得、体会、收获、经验	朱剑林	中南民族大学计算机科学学院
嘉宾	16:40-16:45	全国巡回艺术展第四站 心得、体会、收获、经验	韩喆	河南大学数学与统计学院
嘉宾	16:45-16:50	全国巡回艺术展第一站 心得、体会、收获、经验	周瑞芳	中原工学院数学与信息科学学院
嘉宾	16:50-16:55	全国巡回艺术展第五站 心得、体会、收获、经验	陈宏	西华大学机械学院
嘉宾	16:55-17:00	几何+艺术	孙志斌	中国科学院大学
嘉宾	17:00-17:05	几何+艺术	杨火根	江西理工大学理学院
嘉宾	17:05-17:10	科幻+艺术	陆贝珂	三体电视剧导演
嘉宾	17:10-17:15	数学+艺术	陈鹏	安徽师范大学
嘉宾	17:15-17:20	几何+艺术	刘绍辉	哈尔滨工业大学计算机科学与技术学院
嘉宾	17:20-17:25	内蕴整体几何结构 + 艺术	赵辉	可计算离散整体几何结构实验室
		现场师生交流+合影	师生	太原理工大学数学学院

10 Mesh, Global Geometric Structures, Art Seminar

At the 10th stop of the National Touring Art Exhibition, hosted by the High Magnetic Field Science Center of the Hefei Institutes of Physical Science, Chinese Academy of Sciences, the 8th Grid Roundtable was simultaneously held—a symposium themed "Mesh+ Global Geometric Structures + Art". Relevant scholars were invited to discuss topics including "grids and art" and "global geometric structures and art". The innovative theme of this symposium has greatly promoted research and interest in grid processing and grid generation, as well as advancements in work related to global geometric structures and topological materials.

For interdisciplinary collaborations involving abstract mathematical theories (such as geometric topology), physical materials, and computer algorithms—when sufficient external conditions are lacking—art can serve as a catalyst to initiate direct exchanges among scholars from multiple disciplines. As these exchanges gradually deepen, approaches and methods for multi-disciplinary collaboration will be gradually developed. Typically, different disciplines have their own independent systems in terms of values, work methods, behavioral logic, and tools used, with little overlap between them. However, art is a form that all disciplines can recognize and accept, and a medium through which scholars from diverse fields can find a common language.

Through the National Touring Art Exhibition of Global Geometric Structures we organized, scholars from various disciplines have been able to gather together for preliminary interdisciplinary exchanges—laying the foundation for further interdisciplinary research.

(Figure 92: Poster of the 8th Mesh SquareTable.)

研讨会日程表

主持人：中科院孙志斌老师

1. 全国巡回艺术展第一站心得、体会、收获、经验
   报告人：周瑞芳老师
   中原工学院数学与信息科学学院
   时间：18:00-18:20
2. 艺术和图形学
   报告人：陈鹏老师
   安徽师范大学
   时间：18:20-18:40
3. 几何与艺术
   报告人：杨火根教授
   江西理工大学理学院
   时间：18:40-19:00
4. 全国巡回艺术展第八站心得、体会、收获、经验
   报告人：张东老师
   天津城建大学理学院
   时间：19:00-19:20
5. 网格与艺术
   报告人：刘永财老师
   常州工学院
   时间：19:20-19:40
6. 医学+艺术
   报告人：孟楠老师
   香港大学医学院
   时间：19:40-20:00
7. 整体几何结构艺术的学习心得
   报告人：潘安
   兰州工程师
   时间：20:00-20:20

8.内蕴整体几何结构艺术展
   报告人：赵辉老师
   可计算离散整体几何结构实验室
   时间：20:20-20:40

9.全国巡回艺术展第十站：物理材料+几何结构+艺术
   报告人：杜海峰老师
   中科院合肥强磁场中心
   时间：20:40-21:00

10. 方桌论坛： 整体几何结构+艺术+跨学科
   报告人：全体老师
   时间：21:00-21:40

11 The Role of Art Exhibitions in Industrial Software Technology

The National Touring Art Exhibition of computational discrete Global Geometric Structures goes beyond mere artistic presentation and aesthetic appreciation; instead, it leverages artistic expression to drive the development of relevant engineering technologies. For instance, the geometric topology theories showcased in the exhibition can be applied to address certain challenges currently faced by CAD/CAE/CAM industrial software technologies.

Since 2018, the China Association for Science and Technology (CAST) has continuously organized initiatives to collect and release major scientific and technological challenges. The main forum of the 27th CAST Annual Conference was held in Beijing on July 6, 2025, where major scientific issues, engineering and technical challenges, and industrial technology problems for 2025 were announced. Among these, the first of CAST's Top 10 Engineering and Technical Challenges is titled "Integrated Algorithms and Theories for Design-Simulation-Manufacturing of Complex Models", which states:

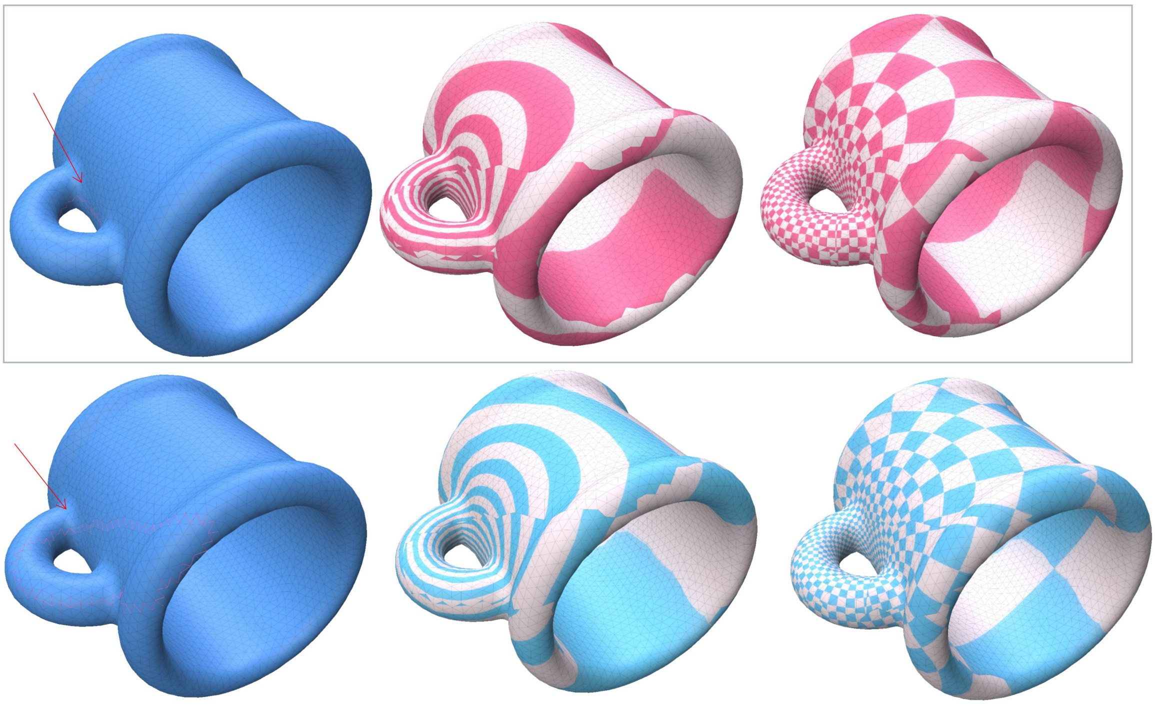
"Breakthroughs and international leadership in China's basic industrial software depend on original innovation and guidance from mathematical theories, requiring systematic and forward-looking comprehensive research layouts. China's CAD/CAE/CAM software development now faces a critical opportunity to catch up with foreign industrial software—this stems from several pain points encountered by international mainstream industrial software in traditional geometric representation and simulation. The geometric kernels of international CAD software took shape in the 1970s and 1980s; over time, entire CAD systems have become extremely complex and cumbersome, making it impossible to replace their underlying data structures and kernels. Consequently, these pain points remain unsolved. As global design and R&D software stands at a historical juncture of upgrading and replacement, it is entirely feasible for China to achieve 'corner overtaking' (a leapfrog advance) over foreign software by addressing these pain points, developing next-generation integrated CAD/CAE geometric kernels, and leveraging the rapid development of China's intelligent manufacturing."

This challenge encompasses multiple dimensions, including the application of mathematical theories, software engineering management, software architecture design, algorithm design, theoretical analysis, problem-solving, code development, engineering collaboration, theory popularization, programming tools, interdisciplinary research, and talent cultivation. We propose a series of innovative approaches that differ from current mainstream perspectives, based on the application of cutting-edge geometric topology theories such as global geometric structures. By overcoming this challenge, we aim to promote the seamless integration of elements including education, scientific research, art, software, code, programming, engineering, science popularization, algorithms, interaction, communication, geometric topology theories, applied mathematics, numerical computation, and solvers—rather than striving solely to achieve the single goal of solving the challenge itself. This approach avoids falling into the "Five-Only" work orientation (over-reliance on academic papers, projects, and other narrow metrics).

The current research status of the engineering and technical challenge "Integrated Algorithms and Theories for Design-Simulation-Manufacturing of Complex Models" is as follows: the specific mathematical theories required to overcome this challenge have not yet been clearly identified. This is distinct from a phase where "the necessary theories are known, and only their research and application are needed." The academic community is currently in an exploratory stage where this core prerequisite remains "unclear." Therefore, our proposal that breakthroughs in this challenge must rely on geometric topology theories related to "global geometric structures" constitutes our primary contribution to solving this engineering and technical challenge.

A brief overview of the solutions and roadmap proposed in this paper is as follows:

(1) We propose that the mathematical theories required for this engineering and technical challenge are cutting-edge geometric topology theories related to global geometric structures—rather than theories already maturely applied in CAD/CAE/CAM industrial software, such as computational geometry and local differential geometry.
(2) We note that the "geometric kernels" (geometric engines) in current industrial software are primarily based on computational geometry theories and technologies, while "mesh engines" are mainly built on local differential geometry theories and technologies.
(3) The pain points caused by "geometric kernels" and "mesh engines" in current international mainstream industrial software—such as issues in isogeometric analysis, spline surface generation, hexahedral mesh generation, T-splines, and finite element computation—cannot be resolved within the framework of existing "geometric kernels." Instead, they require a next-generation "global geometric mesh engine" for solutions.
(4) Specifically, we propose the concept of super-structured quadrilateral meshes with controllable global structural arrangement, based on the application of these cutting-edge geometric topology theories. We further suggest that super-structured quadrilaterals serve as the core to address the aforementioned pain points in CAST’s engineering and technical challenge.
(5) We put forward a next-generation CAD/CAE/CAM industrial software architecture design centered on super-structured quadrilaterals, which emphasizes the equal importance of a new "global geometric mesh engine," "geometric kernel," and "mesh engine." This design aims to enable China’s domestic software to achieve corner overtaking over foreign software.
(6) We will accumulate experience for systematic and forward-looking comprehensive research layouts through initiatives including the open-source meshDGP code, the Joint Research Center for Super-Structured Quadrilateral Meshes, the Grid Roundtable series conferences, study sessions, and academic exchange activities focused on computational discrete global geometric structures.
(7) Taking the solution to this CAST engineering and technical challenge as a driving force, we will advance the interdisciplinary integration and development of multiple fields—including teaching, research, application, science popularization, art, visualization, and communication—in disciplines such as geometric topology theory and application, mesh algorithm design, industrial software development, and mechanical analysis.



(Figure 93: Left: Unstructured triangular mesh; Right: Structured quadrilateral mesh. Created with Geometric.)

Building on the conventional classification systems of structured quadrilateral meshes (Structured Quad Mesh) and regularized quadrilateral meshes, we proposed the new concept of "super-structured quadrilateral meshes (Super Structured Quad Mesh)" with "controllable and designable global quadrilateral arrangement structures" in 2023, and defined its research direction, scope, and core content.

In both academia and industry, mesh generation is generally categorized into three types: unstructured meshes (as shown in the left panel of Figure 22), structured meshes (as shown in the right panel of Figure 22), and hybrid meshes.

Unstructured meshes: Meshes where the surface is composed of triangular elements and the interior is filled with tetrahedral elements.

Structured meshes: Meshes where both the surface and interior are fully filled with quadrilateral elements (surface) and hexahedral elements (interior), respectively.

Hybrid meshes: Meshes composed of a mix of surface elements (e.g., triangles, quadrilaterals, pentagons) and volume elements (e.g., tetrahedrons, hexahedrons).

Structured mesh generation can be further subdivided into four categories: Regular, Semi-Regular, Valence Semi-Regular, and Irregular. However, this subdivision lacks clear and rigorous criteria and is usually roughly defined based on the number of "blocks" (divided regions) in the mesh. Generally, fewer blocks indicate higher mesh regularity. This means that there is currently no algorithm to quantitatively determine the regularity (regular or irregular) of a given structured mesh; in practice, the optimization goal is often "minimizing the number of blocks."

Super-structured quadrilateral meshes—part of the cutting-edge geometric topology theories applied in our proposed solution to the aforementioned challenge—are featured in the exhibits of this national touring art exhibition.

The research direction of super-structured quadrilateral meshes also helps address several pain points in the engineering and technical challenge of "Integrated Algorithms and Theories for Design-Simulation-Manufacturing of Complex Models", including isogeometric analysis, spline surface generation, hexahedral mesh generation, T-spline generation, and surface intersection computation.

Currently, scholars in the research field of CAD/CAE/CAM industrial software technology mainly come from disciplines such as mechanics, mechanical engineering, and computer science. Few experts and scholars in these disciplines have a grasp of cutting-edge geometric topology theories such as "global geometric structures." Even within the mathematics discipline, the number of scholars who master these theories is limited. Furthermore, scholars who can apply these cutting-edge geometric topology theories—especially in the context of CAD/CAE/CAM industrial software — are even scarcer. The shortage of interdisciplinary talents proficient in applying geometric topology theories is one of the reasons why this engineering technology has become a pressing challenge.

Therefore, through this national touring art exhibition, teachers and students can gain a better understanding of these mathematical theories, thereby cultivating talents for solving industrial software technology challenges.

12 The Significance of the Art Exhibition

(1) Building on the conventional classification systems of structured quadrilateral meshes (Structured Quad Mesh) and regularized quadrilateral meshes, we proposed the new concept of "super-structured quadrilateral meshes (Super Structured Quad Mesh)" with "controllable and designable global quadrilateral arrangement structures" in 2023, and defined its research direction, scope, and core content.

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(6) Structured mesh generation can be further subdivided into four categories: Regular, Semi-Regular, Valence Semi-Regular, and Irregular. However, this subdivision lacks clear and rigorous criteria and is usually roughly defined based on the number of "blocks" (divided regions) in the mesh. Generally, fewer blocks indicate higher mesh regularity. This means that there is currently no algorithm to quantitatively determine the regularity (regular or irregular) of a given structured mesh; in practice, the optimization goal is often "minimizing the number of blocks."

(7) Super-structured quadrilateral meshes—part of the cutting-edge geometric topology theories applied in our proposed solution to the aforementioned challenge—are featured in the exhibits of this national touring art exhibition.

(8) The research direction of super-structured quadrilateral meshes also helps address several pain points in the engineering and technical challenge of "Integrated Algorithms and Theories for Design-Simulation-Manufacturing of Complex Models", including isogeometric analysis, spline surface generation, hexahedral mesh generation, T-spline generation, and surface intersection computation.

(9) Currently, scholars in the research field of CAD/CAE/CAM industrial software technology mainly come from disciplines such as mechanics, mechanical engineering, and computer science. Few experts and scholars in these disciplines have a grasp of cutting-edge geometric topology theories such as "global geometric structures." Even within the mathematics discipline, the number of scholars who master these theories is limited. Furthermore, scholars who can apply these cutting-edge geometric topology theories—especially in the context of CAD/CAE/CAM industrial software—are even scarcer. The shortage of interdisciplinary talents proficient in applying geometric topology theories is one of the reasons why this engineering technology has become a pressing challenge.

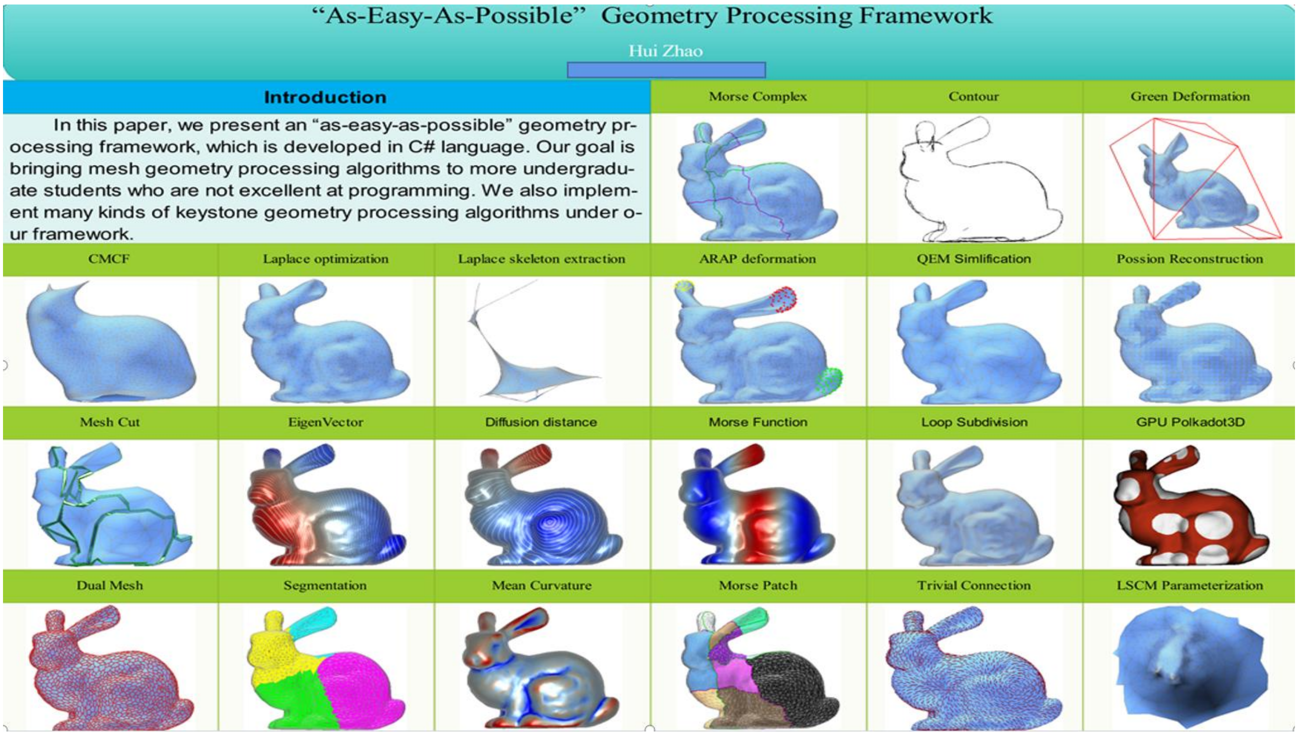
(10) Therefore, through this national touring art exhibition, teachers and students can gain a better understanding of these mathematical theories, thereby cultivating talents for solving industrial software technology challenges.

13 The Technology for Art Exhibition

13.1 网格处理软件 meshDGP

本次全国巡回艺术展的图片是基于我们在网格编程、几何拓扑理论、工业技术应用等方面的研究。自 2007 年起，我们自主研发了网格处理软件 meshDGP。该软件包含数十万行代码，采用 C# 语言开发，集成了数十至数百种各类网格处理算法，涵盖三维模型简化、细化、变形、参数化等功能，如下图所示。

据粗略统计，目前全国已有约 4-5 万名理工科师生及工程师使用该软件进行学习、科研与教学活动。近十年来，它被广泛应用于计算机图形学、力学分析、机械工程、微分几何、虚拟现实、生物材料、CAD/CAE、有限元分析、工业软件等多个学科领域的教学、科研与工程实践中。



（图 94：各种建模算法展示，meshDGP 做图。）

（Figure 94：The demonstrations of many modling algorithms，generated by meshDGP。）

根据实名注册统计当前学习和使用 meshDGP 代码的跨学科各行业的师生、工程师有近千人，他们的研究方向有：机械工程、建筑设计、数值分析、航空宇航科学与技术、应用数学、GIS 与 BIM、结构工程、土木工程、设计与媒体工程、建筑与土木工程、建筑学、计算数学、水利水电工程、智能建造、太阳物理、计算机视觉、计算机科学技术、网格生成与应用、电磁计算、基础数学、偏微方程数值解、微分几何、应用数学、测控、共形几何、图形学、控制科学与工程、工程力学、偏微分方程、网格处理、等几何分析工程应用、随机分析、光学工程、高性能计算、无线电物理、智能计算、软件工程、自动控制、计算流体动力学、有限元、信号与信息处理、地质建模、物理、航空宇航工程、软件工程、力学、生物、多相流流固耦合高性能计算、电路与系统、医学图像计算、人工智能、应用统计、控制理论与控制工程、结构设计、车辆工程、应用物理学、机械创新结构设计、生物力学、医学图像分析、建筑形态量化研究、工程热物理、核能科学与工程、风力机空气动力学、机械设计及理论、材料科学、几何处理、广义有限元、飞行器设计、准晶，分形几何，图嵌入、结

构拓扑优化、通信、电子科学与技术、流体力学、电磁场、计算机安全、航天工程与力学系等等。

meshDGP 开源代码相关的有赵辉老师撰写的五本教材，通过五本教材和代码对照，很多师生、工程师得以在最短时间内跨学科入门网格处理的相关算法。

1. 计算机图形学：三维模型处理算法初步（C sharp 版本），赵辉等. 海洋出版社，2014。
2. 计算机图形学：OpenGL 三维渲染（C sharp 版本），赵辉等. 海洋出版社，2016。
3. 三维模型参数化算法: 理论和实践（C sharp 版本），赵辉等. 电子工业出版社，2017。
4. 三维模型变形算法 理论和实践（C sharp 版本），赵辉等. 电子工业出版社，2017。

GLSL 渲染编程基础与实例（C sharp 版本），赵辉等. 电子工业出版社，2017。

（图 95：五本教材。）

13.2 基于 meshDGP 的微分几何教学

CAD-CAE-CAM 工业软件的核心数学理论涵盖计算几何、微分几何等多个几何分支。尽管计算几何相关理论已较为普及，但微分几何在工科领域的应用与教学仍存在明显短板。全国开设微分几何课程的非数学专业数量有限，降低该学科对非数学专业学生的学习门槛尤为重要。事实上，多数专业学生常因微分几何中复杂的公式与抽象的数学语言感到枯燥晦涩，理解难度较大。

为此，在多所高校领导的支持下，我们联合中原工学院理学院、河南大学、昆明理工大学、北京邮电大学、烟台大学、河海大学、中国科学院大学等院校的教师，发起了“基于 meshDGP 的微分几何可视化教学”创新教改研讨交流会，旨在提升学生对微分几何的学习兴趣，助力其深入理解学科内涵。依托 meshDGP 平台，我们将微分几何中的概念与定理以可视化方式呈现，帮助学生快速理解抽象的几何概念、定理及公式。

（图 96：微分几何研讨会海报。）

13.3 系列网格方桌会议

和全国巡回艺术展紧密相关的是网格方面的研究，网格研究并非纯粹的抽象理论探索，而是融合几何拓扑理论应用、数值计算、代码实现、可视化渲染等多维度的综合性研究。正是基于推动这一领域发展的目标，我们创立和开展了系列网格方桌会议。

值得一提的是，该系列会议之所以命名为 “方桌” 而非常见的 “圆桌”，核心原因在于我们针对中国科学领域这一难题提出的解决方案：“超结构化四边形网格”。“超结构化四边形网格”就是研究利用内蕴的各种整体几何结构等前沿微分几何拓扑理论，来对曲面上划分四边形格子。四边形的形态与 “方” 的意象高度契合，因此 “方桌” 这一命名更能精准呼应我们的研究内核。

目前，网格方桌系列会议已经举办了八届，每一届研讨一个和网格研究相关的不同主题，例如代码编程、工业开发、可视化渲染、微分几何教学、优化问题、求解器、计算机图形学交叉、论文写作风格、网格艺术等。每一届探讨会参加的有全国各地的不同学校不同转的老师、博士生、企业的工程师。这个系列会议仍旧在持续进行中，后续还有更多的和网格相关的研讨题目。

1. 第一届 网格方桌会议：代码编程层面
   天津城建大学理学院承办 开幕词：贾国治院长 主办人：张东老师。
2. 第二届 网格方桌会议：CADCAE 小软件大开发层面
   安徽师范大学物理与电子信息学院承办 开幕词：张爱清院长 主办人：陈鹏老师
3. 第三届 网格方桌会议：可视化渲染层面

江西理工大学理学院承办　开幕词：　徐中辉院长　主办人：　杨火根老师

4. 第四届 网格方桌会议：面向代码的微分几何可视化学习和教学

   河南大学数学与统计学院承办　开幕词：唐恒才院长　主办人：　韩喆老师

5. 第五届 网格方桌会议：优化问题、线性系统、求解器

   北京工业大学数学统计与力学学院承办　开幕词：黄秋梅院长　主办人：　诸葛昌靖老师

6. 第六届 网格方桌会议：　计算机图形学+其他学科交叉融合

   中原工学院数学与信息科学学院承办　开幕词：闫振亚院长　主办人：周瑞芳老师

7. 第七届网格方桌会议：网格+可计算整体几何结构方面论文写作风格评析

   常州工学院理学院院长承办　开幕词：陈荣军院长　主办人：刘永财老师

8. 第八届网格方桌会议：网格+整体几何结构+艺术

   中国科学院合肥物质科学研究院强磁场科学中心承办　开幕词：杜海峰主任。

（图 97：第五届网格方桌会议。）

13.4 举办 meshDGP 学习会

很多师生对艺术展的技术感兴趣，因此我们陆续举办了 80 余场实名专题学习会。通过这些学习会，来自全国多所院校、不同研究方向的 80 余位老师、学生及工程师，得以在工作之余高效掌握艺术展所使用技术入门要点，为深入研究奠定了基础，部分学习题目和学习人如下所示。

题目	学习人
MeshDGP里的Morse函数	潘安 兰州
MeshDGP里的特征向量计算展示	瞿悦呈 北航计算机学院博士生
MeshDGP里面的QEM简化算法展示和代码流程	冯骊骁 老师 重庆科技大学计算机学院
MeshDGP里的向量场展示和代码流程【1/3】	赵晴 中原工学院数学学院 本科生
MeshDGP里的Calabi-Yau曲面编程	叶然然 本科生 中原工学院数学与信息科学学院数学与应用数学
MeshDGP里面的参数化效果图展示、代码概述：频谱，ABF算法篇章【1/2】	殷思麒 湖南大学机械学院 博士生
MeshDGP里的OpenGL展示和代码流程【1/2】	康亚卓 北京 中机认检
MeshDGP里的拉伸能量算法展示、代码概述【1/4】	丁弘晖 人大统计学博士生
MeshDGP里的OpenGL展示和代码流程Ⅱ	潘安 兰州
MeshDGP里的GLSL效果展示和代码流程	祁桐 北京构力科技有限公司
四边形六面体网格生成之Polycube Shap Space	瞿悦呈 北航计算机学院博士生
基于Streching Energy的变形实验展示	潘安 兰州
meshDGP剪影等操作的初步认识和后续学习目标	杜志楠 中国航空制造技术研究院
meshDGP里的GLSL渲染二刷	胡志林 博士 清华精密仪器系毕业
meshdgp里面的高斯曲率，平均曲率，欧拉示性数，亏格，点，边，面展示和	朱施恩 威斯康星-麦迪逊大学 UW madison 统计专业 研究生
线性代数之——MeshDGP里的OpenGL之矩阵变换展示和代码流程	孙竞争 广东省科学院智能制造研究所
meshDGP软件架构和设计模式	康亚卓 潘安 彭贯军 黄锦龙 祁桐 vs 曾珂 西安建筑大学
meshDGP里如何实现样条曲面【1/2】	宋洋 博士 犹他大学 师从样条前辈教授
meshDGP里的骨骼动画和蒙皮展示与代码流程	肖智文 博士 清华航院
meshDGP里如何实现细分subdivision曲面【1/2】	杨火根 教授 江西理工大学
meshDGP里的贝塞尔曲线展示和代码流程【1/3】	银润龙 老师 忻州师范学院数学系
meshDGP里的简化光滑等操作和代码流程【1/3】	陆云桥 工程师 上海正雅齿科
meshDGP里的梯度，拉普拉斯与双调和操作和代码【1/2】（PPT代替）	刘亚雄 湖南
meshDGP里的三角形、四边形网格生成展示和代码流程【1/2】	王少阳 工程师 制造行业
meshDGP里的B-Spline曲线展示和代码流程【1/3】	黄锦龙 工程师 深圳 激光行业
meshDGP里的三维模型点边面的增加删除操作展示和代码流程【1/2】	陈鹏 老师 安徽师范大学物理与电子信息化学院
meshDGP里的三维模型补洞、边界操作展示和代码流程【1/2】	王卫 中学老师 世青国际学校
meshDGP里的刚体转动 通过OpenGL【1/3】	姬振华 老师 郑州航院力学
MeshDGP里面的5,6,7度数操作展示和代码流程	李林 老师 太原科技大学机械工程学院
meshDGP里的matlab调用	刘宇辉 中南大学 老师 人工智能方向
meshDGP里的网格切割操作展示和代码流程	杨佩栎 北航电子信息工程 研究生
三维模型参数化扭曲度量及和谐参数化理论代码展示（无视频 使用PPT）	李鹏 北航
meshDGP里的补洞挖洞操作和代码流程	詹国峰 工程师 三维技术相关行业
MeshDGP里的ArcBall里面的（线性变换）原理和选择功能展示和代码流程	段育鹏 哈尔滨工程大学 博士生
meshDGP里的人机交互和UI	刘婧媛 东京大学五十岚 健夫Takeo Igarashi教授实验室博士后 香港科技大学visgraph lab博士
meshDGP里的线性代数50个应用之一	诸葛昌靖 北京工业大学数学学院 线性代数老师
MeshDGP里的三维模型特征线条抽取算法功能展示和代码流程	刘永才 老师 常州工学院理学院
meshDGP里的possion重建效果展示和对照代码	曾珂 教授 西安建筑大学
meshDGP里的法流操作展示和代码对照	古鹏举 博士 贵州大学
meshDGP的possion重建操作展示和对照代码	王川川 工程师 宁波
meshDGP里的polycube变形	陈晨曦 博士 南洋理工大学
meshDGP里的GLSL渲染	孟楠 教授 香港大学医学院
meshDGP概览和研究生指导辅助工具	孙志斌 教授 博导 中国科学院大学 中国科学院国家空间科学中心
meshDGP里的三维模型分段算法展示	邵弘毅 博士 航发商发上海
meshDGP里的贴图和featureLine学习	伍世聪 工程师 北京+苏州

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(3) North China Electric Power University

(4) Henan University

(5) Xihua University

(6) Renmin University of China

(7) South-Central Minzu University

(8) Tianjin Chengjian University

(9) Taiyuan University of Technology

(10) High Magnetic Field Science Center, Hefei Institutes of Physical Science, Chinese Academy of Sciences

(11) Jiangxi University of Science and Technology

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